Chapter 1: Introduction

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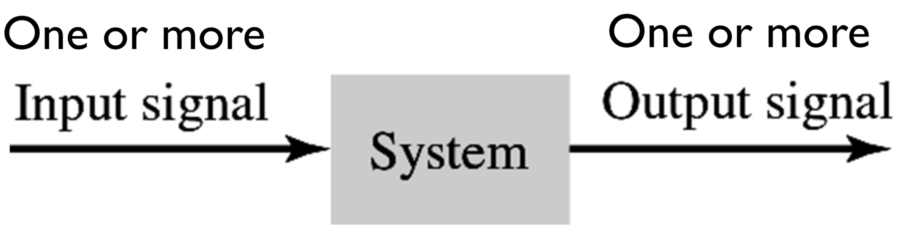
Outline

- What is a signal?
- What is a system?
- Overview of specific systems
- Classification of signals
- Basic operations on signals
- Elementary signals
- Systems viewed as interconnections of operations
- Properties of systems
- Noises
- Theme example
- Exploring concepts with MATLAB

Introduction

- What is a signal?
  - A signal is formally defined as a function of one or more variables that conveys information on the nature of a physical phenomenon.
- What is a system?
  - A system is formally defined as an entity that manipulates one or more signals to accomplish a function, thereby yielding new signals.



Overview of Specific Systems

Example 1: Communication Systems

Figure 1.2: Elements of a communication system. The transmitter changes the message signal into a form suitable for transmission over the channel. The receiver processes the channel output (i.e., the received signal) to produce an estimate of the message signal.

- Analog communication systems
  - Modulator (AM, PM, FM) → Channel → Demodulator
- Digital communication systems
  - (Sampling+Quantization+Modulation+Coding) → Channel → (Reversed Function)
- Wireless/Wired Channel
  - Noise

Specific System Example 2

- Feedback Control Systems

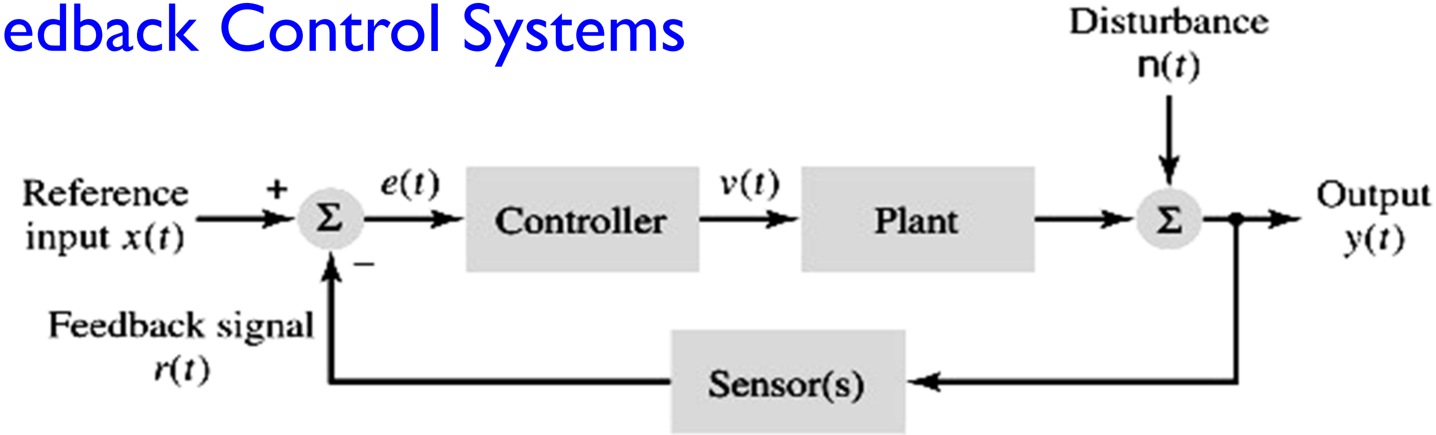
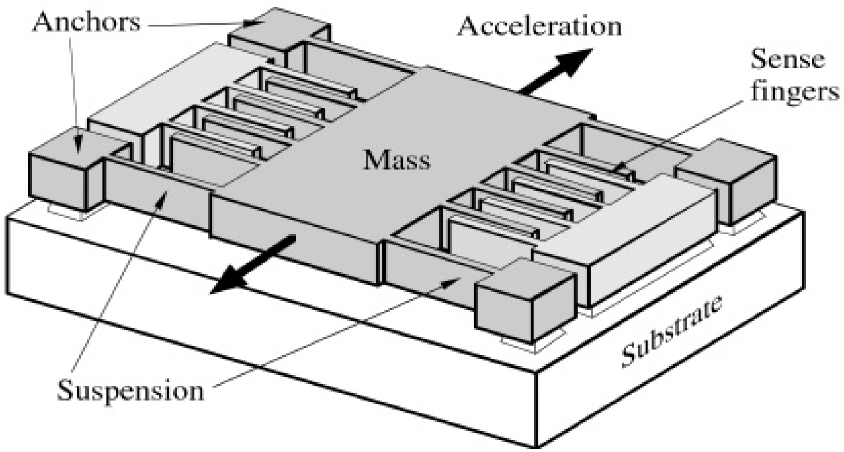


Figure 1.4 Block diagram of a feedback control system. The controller drives the plant, whose disturbed output drives the sensor(s). The resulting feedback signal is subtracted from the reference input to produce an error signal *e*(*t*), which, in turn, drives the controller.

- Response and Robustness
  - Single-input, single-output (SISO) system
  - Multiple-input, multiple-output (MIMO) system

Specific System Example 3

- Micro-electro-mechanical Systems (MEMS)
  - Merging mechanical systems with microelectronic control circuits on a silicon chip.



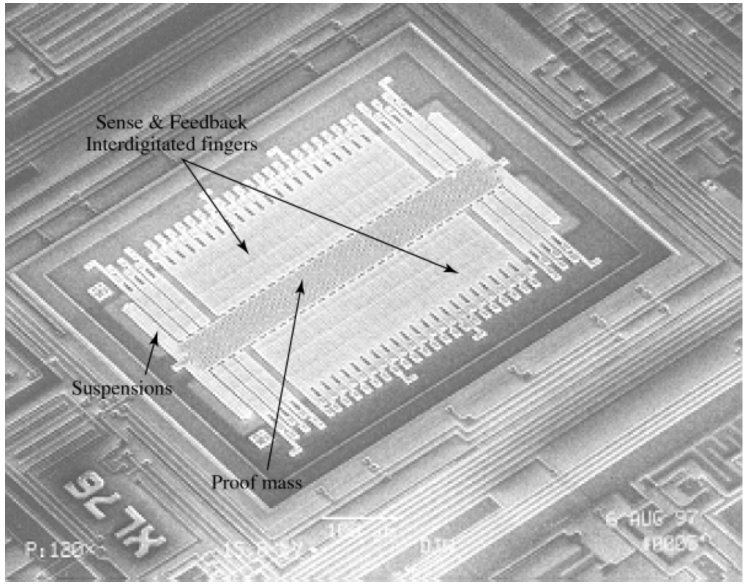


Figure 1.6 (Taken from Yazdi et al., *Proc. IEEE*, 1998)
(a) Structure of lateral capacitive accelerometers.
(b) SEM view of Analog Device's ADXLO5 surface-micromachined polysilicon accelerometer.

Specific System Example 4

- Remote Sensing
  - The process of acquiring information (detecting and measuring the changes) about an object of interest without being in physical contact with it
  - Types of remote sensor
    - Radar sensor
    - Infrared sensor
    - Visible/near-infrared sensor
    - X-ray sensor

Figure 1.7 Perspectival view of Mount Shasta (California), derived from a pair of stereo radar images acquired from orbit with the Shuttle Imaging Radar. (Courtesy of Jet Propulsion Lab.)

Specific System Example 5

- Biomedical Signal Processing

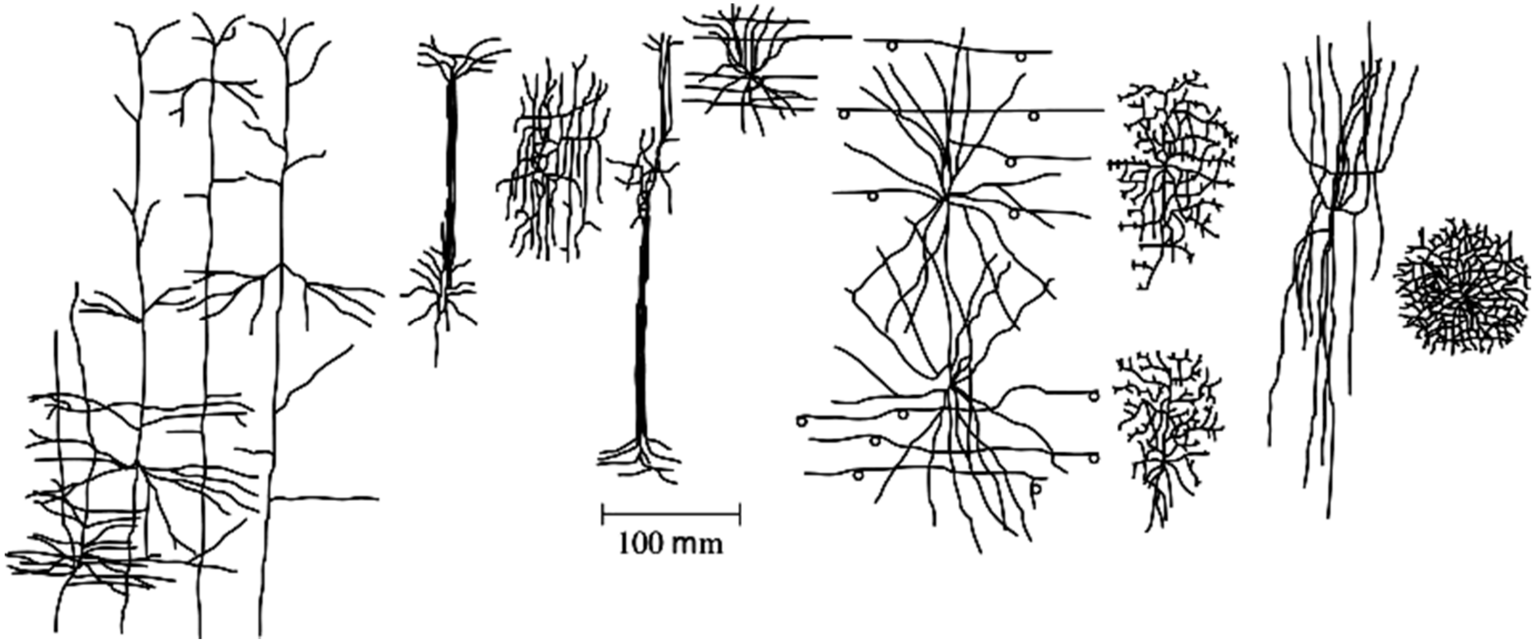


Figure 1.8 Morphological types of nerve cells (neurons) identifiable in monkey cerebral cortex, based on studies of primary somatic sensory and motor cortices. (Reproduced from E. R. Kande, J. H. Schwartz, and T. M. Jessel, *Principles of Neural Science*, 3d ed., 1991; courtesy of Appleton and Lange.)

Specific System Example 5

- Many biological signals (found in human body) is traced to the electrical activity of large groups of nerve cells or muscle cells

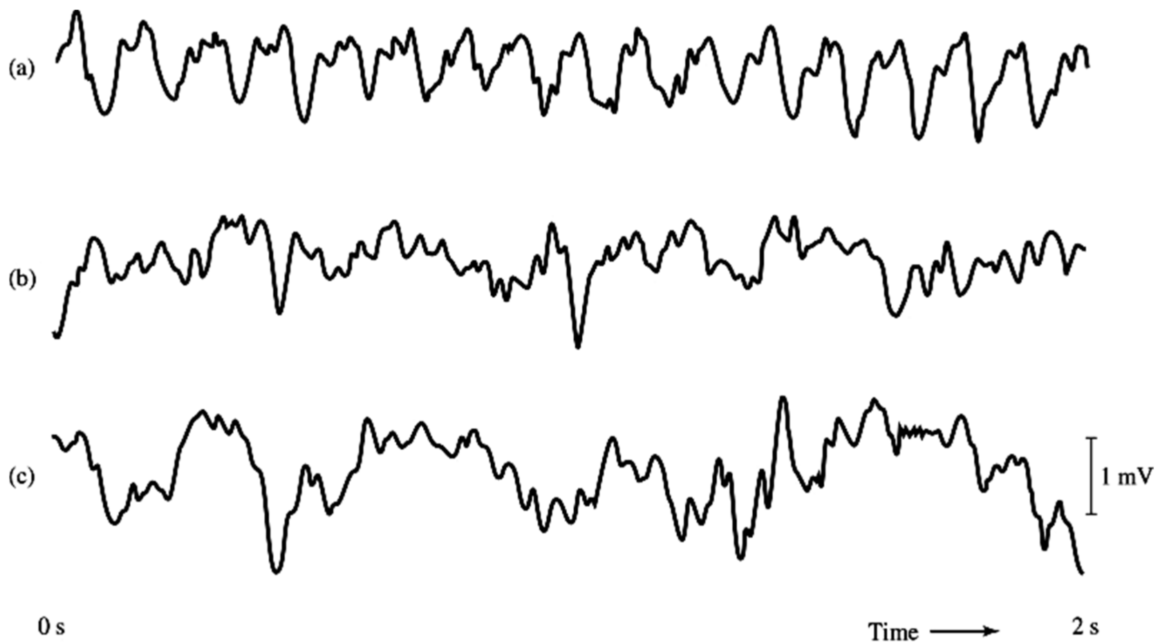
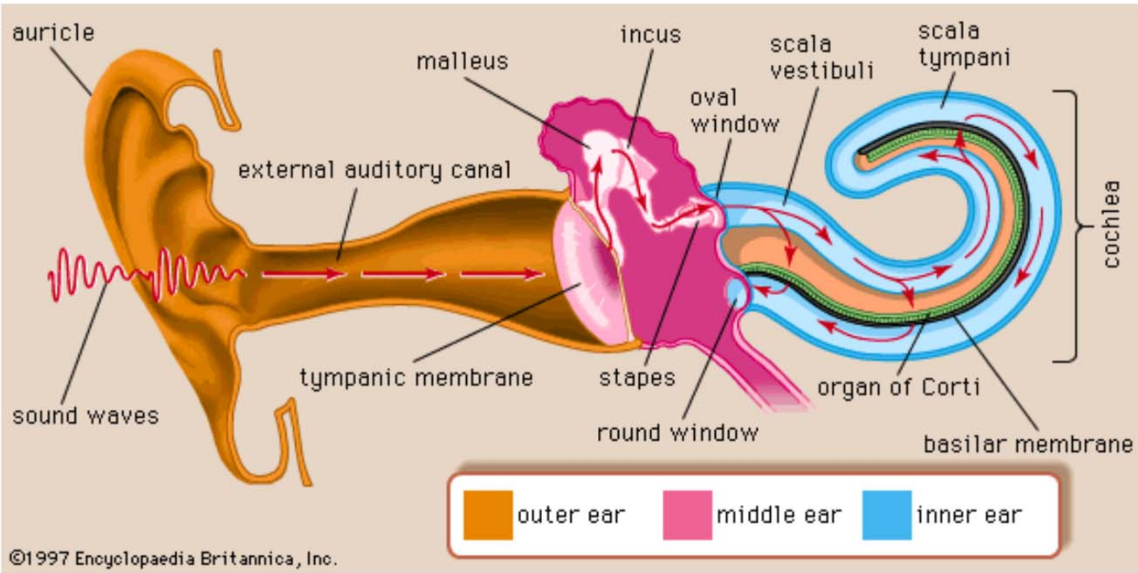


Figure 1.9

The traces shown in (a), (b), and (c) are three examples of EEG signals recorded from the hippocampus of a rat. Neurobiological studies sugge`st that the hippocampus plays a key role in certain aspects of learning and memory.

Specific System Example 6

- **Auditory System**
  - The three main parts of the ear



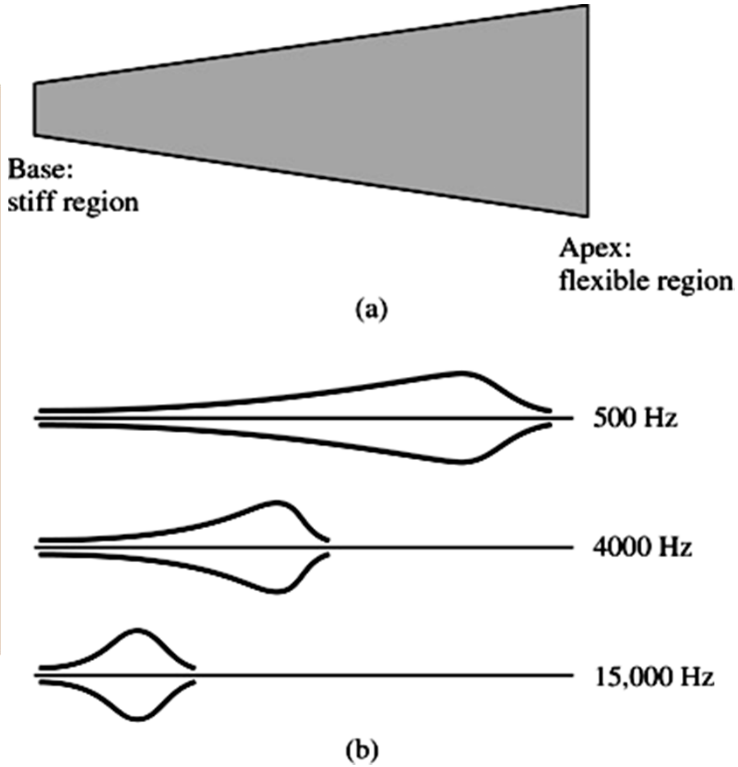


Figure 1.10
(a) In this diagram, the basilar membrane in the cochlea is depicted as if it were uncoiled and stretched out flat. (b) This diagram illustrates the traveling waves along the basilar membrane.

Overview of Specific Systems

- Analog versus Digital Signal Processing
  - Continuous-time approach
    - Natural way
    - Analog circuit elements: resistors, capacitors, inductors, AP, and diodes
  - Discrete-time approach
    - More complex and artificial way
      - Sampling (ADC) and Reconstruction (DAC)
    - Digital circuit elements: adder, shifter, multiplier, and memory
    - Flexibility
    - Repeatability

Classification of Signals

- We restrict our attention to one-dimensional signals only
  - 1. Continuous-time and discrete-time signals
    - Continuous-time signals:
      - Real-valued or complex-valued function of time: $x(t)$
    - Discrete-time signals:
      - A time series: $\{x[n] = x(nT_s), n = 0, \pm 1, \pm 2, \ldots\}$
      - T_s: sampling period

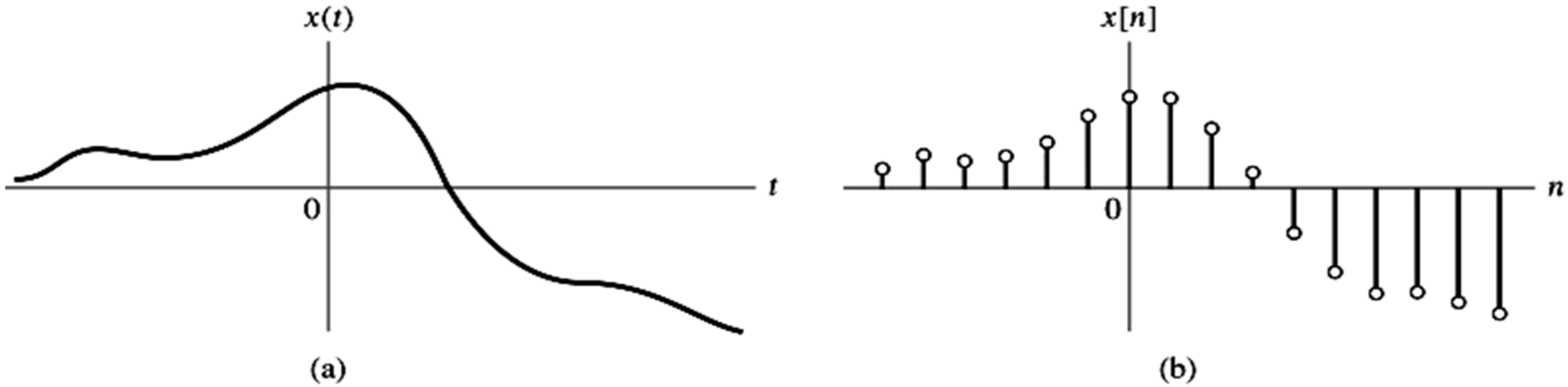
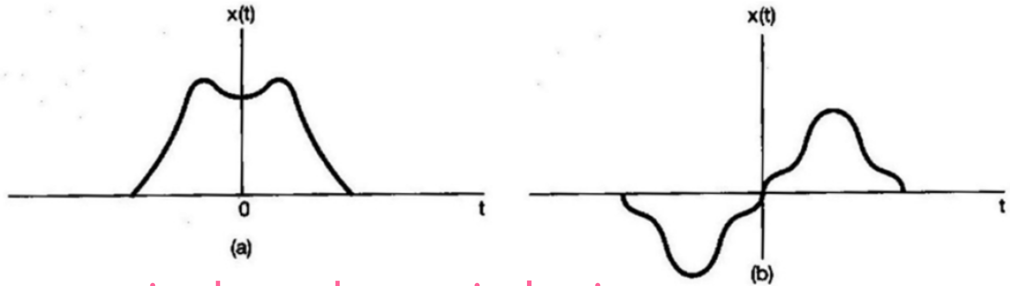


Figure 1.12
(a) Continuous-time signal $x(t)$. (b) Representation of $x(t)$ as a discrete-time signal $x[n]$.

Classification of Signals

- 2. Even and odd signals
- For real-valued, continuous (or discrete) signal
  - Even signals: $x(-t) = x(t), \forall t$
  - Odd signals: $x(-t) = -x(t), \forall t$

Symmetric about the origin



Symmetric about the vertical axis

- Example 1.1: Even or odd signal?

$$x(t) = \begin{cases} \sin\left(\dfrac{\pi t}{T}\right), & -T \le t \le T \\ 0 \quad , & \text{otherwise} \end{cases}$$

Even-Odd Decomposition of Signals

- An arbitrary signal $x(t) = x_e(t) + x_o(t)$ where $x_e(-t) = x_e(t)$, $x_o(-t) = -x_o(t)$

$$x(-t) = x_e(-t) + x_o(-t) = x_e(t) - x_o(t)$$

$$x_e = \frac{1}{2}[x(t) + x(-t)] \quad (1.4)$$

$$x_o = \frac{1}{2}[x(t) - x(-t)] \quad (1.5)$$

- Example 1.2
  - Even-odd decomposition of $x(t) = e^{-2t} \cos t$

Even component:

$$x_e(t) = \frac{1}{2}(e^{-2t} \cos t + e^{2t} \cos t) = \cosh(2t)\ \cos t$$

Odd component:

$$x_o(t) = \frac{1}{2}(e^{-2t} \cos t - e^{2t} \cos t) = -\sinh(2t) \cos t$$

Conjugate Symmetric Complex Signals

- A complex-valued signal $x(t)$ is *conjugate symmetric* if its real part is even and its imaginary part is odd.

Proof:

Let $x(t) = a(t) + jb(t)$

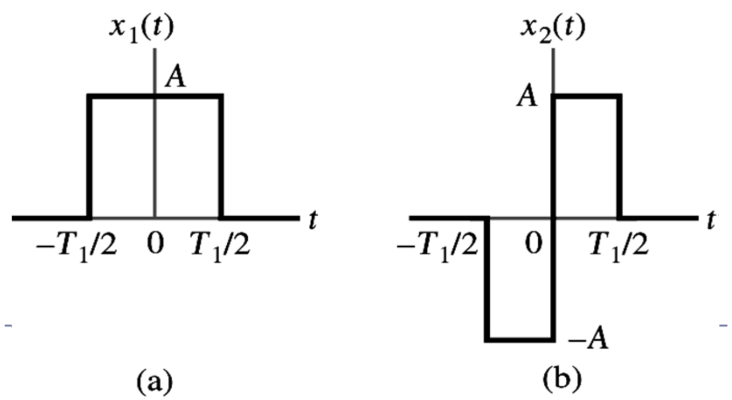
$$\Rightarrow x^*(t) = a(t) - jb(t)$$

$$\Rightarrow a(-t) + jb(-t) = a(t) - jb(t)$$

$$\Rightarrow \quad a(-t) = a(t), \quad b(-t) = -b(t)$$

- Example 1.2
  - A conjugate symmetric signal, where its real part is depicted in Fig. 1.13(a) and the imaginary part is in Fig. 1.13(b)

Figure 1.13



Classification of Signals

- 3. Periodic and nonperiodic signals
  - Periodic continuous-time signal: $x(t+T) = x(t), \forall t$
    - Clearly, $T=T_0, 2T_0, 3T_0, \ldots$.. Then, T_0 is called fundamental period and $2T_0, 3T_0, \ldots$ are harmonic
    - The reciprocal of the fundamental frequency is called frequency $f = \frac{1}{T_0}$
    - And, the angular frequency is defined by $\omega = 2\pi f$
  - Nonperiodic signal: There is no finite T such that $x(t+T) = x(t), \forall t$
  - Example: (a) periodic and (b) nonperiodic continuous-time signals

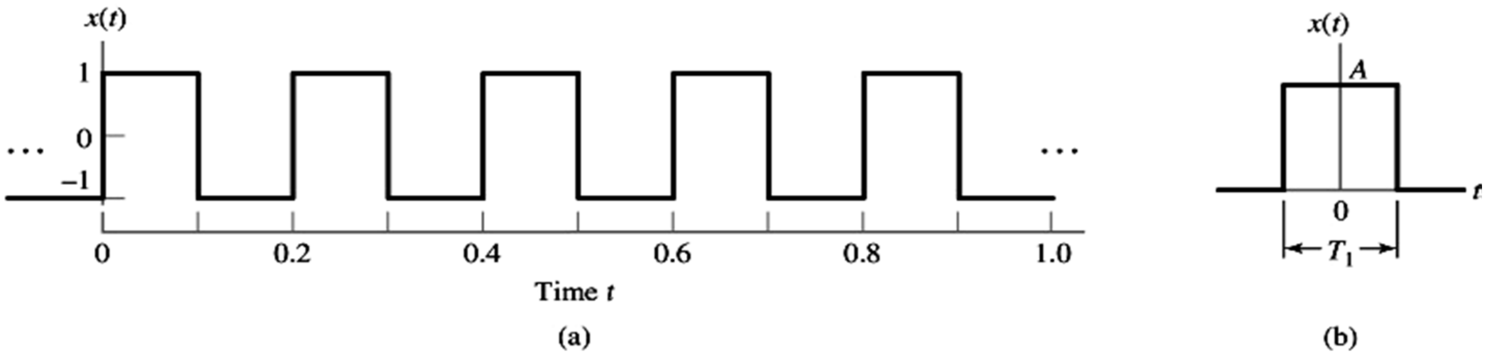
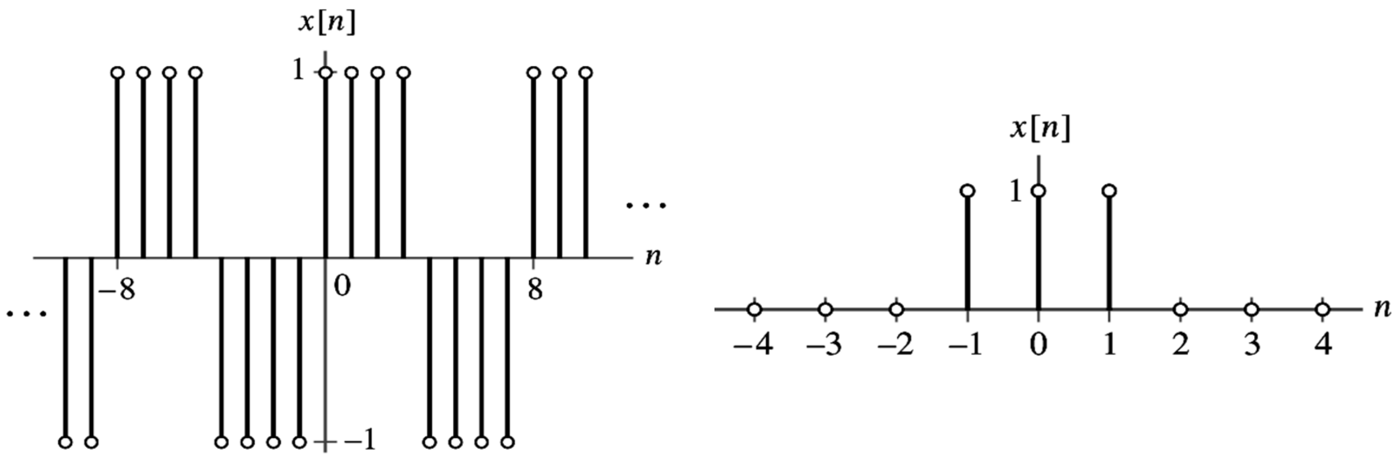


Figure 1.14 (a) Square wave with amplitude $A = 1$ and period $T = 0.2$s. (b) Rectangular pulse of amplitude A and duration T_1.

Classification of Signals

- 3. Periodic and nonperiodic signals
  - Periodic discrete-time signal: $x[n+N] = x[n]$, for integer n
    - N is a positive integer
    - The smallest integer N is called the fundamental period of $x[n]$
    - The fundamental angular frequency is defined by $\Omega = 2\pi/N$
  - Nonperiodic signal: There is no finite N such that $x[n+N] = x[n]$, for integer n
  - Example: periodic and nonperiodic discrete-time signals



Classification of Signals

- 4. Deterministic signals and random signals
  - A *deterministic signal* is a signal about which there is no uncertainty with respect to its value at any time.
    - sin(t), cos(t), …
  - A *random signal* is a signal about which there is uncertainty before it occurs.
    - noise, stock price index, …
- 5. Energy signals and power signals
  - The total energy of the continuous-time signal $x(t)$ is defined by

$$E = \lim_{T\to\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt = \int_{-\infty}^{\infty} x^2(t)dt \quad (1.15)$$

  - The power of the signal $x(t)$ is defined by the time-averaged of (1.15)

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt \quad (1.16) \qquad \text{or} \qquad P = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt \quad (1.17)$$

For nonperiodic signal — For periodic signal

Energy Signals and Power Signals

- For discrete-time signal $x[n]$, (1.15)–(1.17) become to

$$E = \sum_{n=-\infty}^{\infty} x^2[n] \quad (1.18)$$

$$P = \lim_{n\to\infty} \frac{1}{2N} \sum_{n=-N}^{N} x^2[n] \quad (1.19)$$

$$P = \frac{1}{N} \sum_{n=0}^{N-1} x^2[n] \quad (1.20)$$

- Energy signals: iff (if and only if) $0 < E < \infty$
- Power signals: iff $0 < P < \infty$

- The energy and power classifications of signals are mutually exclusive
- An energy signal has zero time-averaged power
- A power signal has infinite energy
- The periodic signals and random signals are usually power signals
- The deterministic, nonperiodic signals are usually energy signals

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Basic Operations on Signals

- Operations performed on dependent variables
  - Amplitude scaling

    $y(t) = cx(t)$ Amplifier, resistor, …

    $y[n] = cx[n]$

  - Addition

    $y(t) = x_1(t) + x_2(t)$ Audio mixer, …

    $y[n] = x_1[n] + x_2[n]$

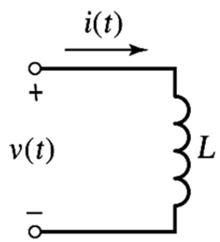
  - Multiplication

    $y(t) = x_1(t)x_2(t)$ Frequency mixer, AM signal,…

    $y[n] = x_1[n]x_2[n]$

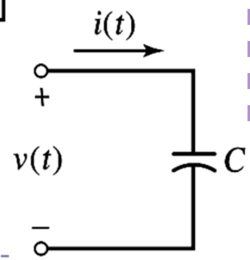
  - Differentiation

    $$y(t) = \frac{d}{dt}x(t)$$ Inductor, …



  - Integration

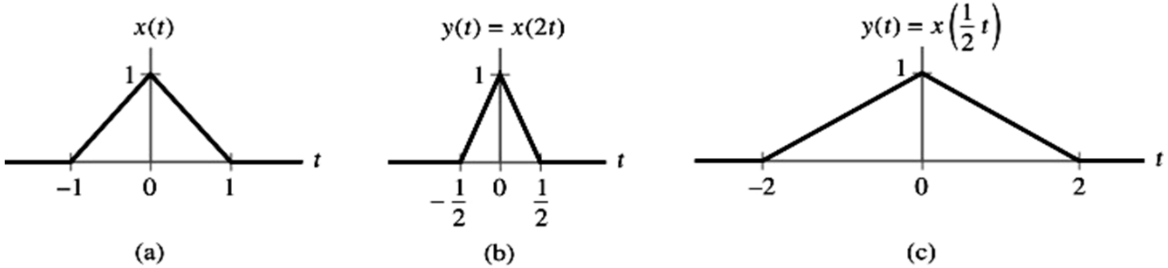
    $$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$ Capacitor, …



Basic Operations on Signals

- Operations performed on the independent variable
  - Time scaling

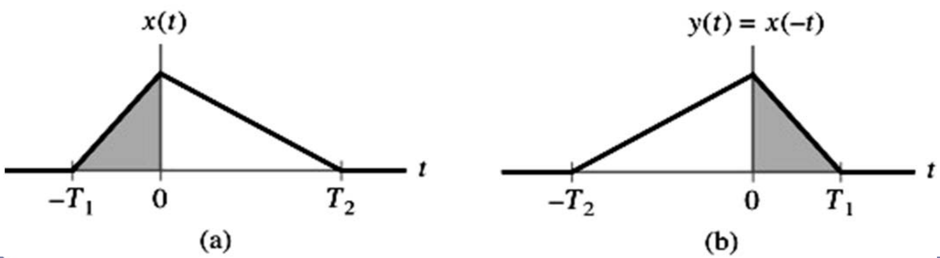
$$y(t) = x(at) \Rightarrow \begin{cases} a > 1, & \text{compressed} \\ 0 < a < 1, & \text{expanded} \end{cases}$$



$$y[n] = x[kn], \quad k > 0$$

  - Reflection (about t=0)

$$y(t) = x(-t)$$



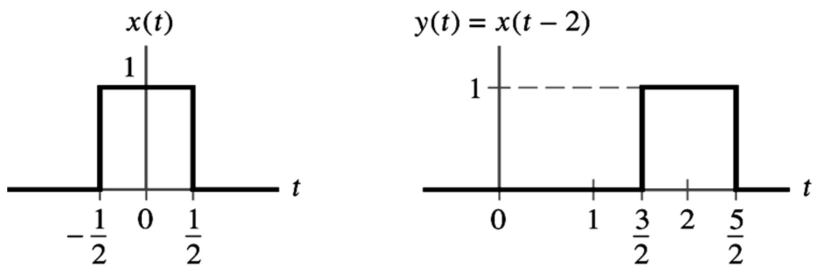
$$y[n] = x[-n]$$

Basic Operations on Signals

- Time shifting

$$y(t) = x(t - t_0) \Rightarrow \begin{cases} t_0 > 0, & \text{right shift} \\ t_0 < 0, & \text{left shift} \end{cases}$$

$$y[n] = x[n - m]$$



(a) (b)

Basic Operations on Signals

- Precedence rule for time shifting and time scaling
  - A combination of time shifting and time scaling operations

$$y(t) = x(at - b)$$

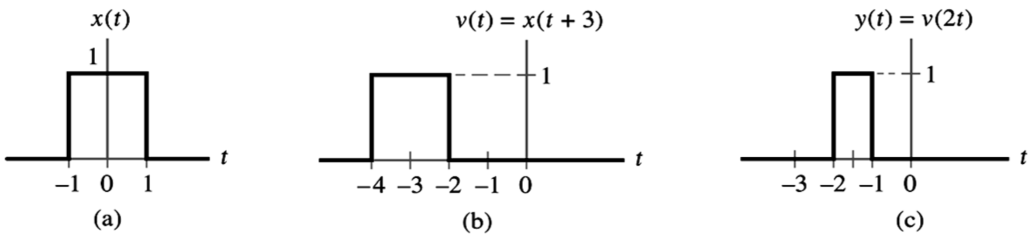
  - The operations must be performed in the correct order
    - The scaling operation always replaces t by at
    - The shifting operation always replaces t by $t-b$

$\Rightarrow$ time-shifting operation is performed first

$$v(t) = x(t-b)$$

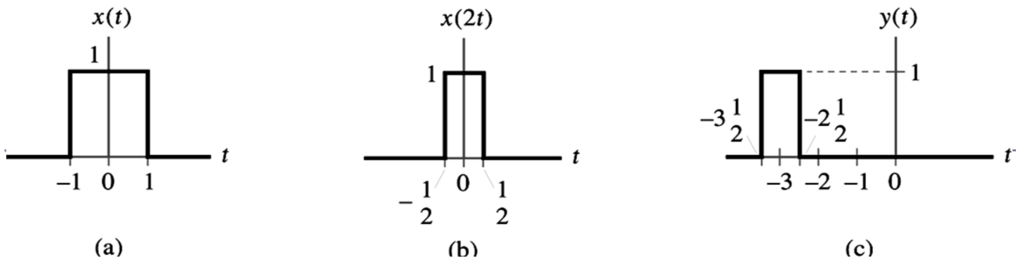
$$y(t) = v(at) = x(at-b)$$

  - Example 1.5 $\quad y(t) = x(2t+3)$



$y(0) = x(3)$

$y(\frac{-3}{2}) = x(0)$



$x(2t+3) \neq x(2(t+3))$

- Example 1.6

A discrete-time signal is defined by $x[n] = \begin{cases} 1, & n = 1,2 \\ -1, & n = -1,-2 \\ 0, & n = 0 \text{ and } |n| > 2 \end{cases}$

Find $y[n] = x[2n + 3]$.

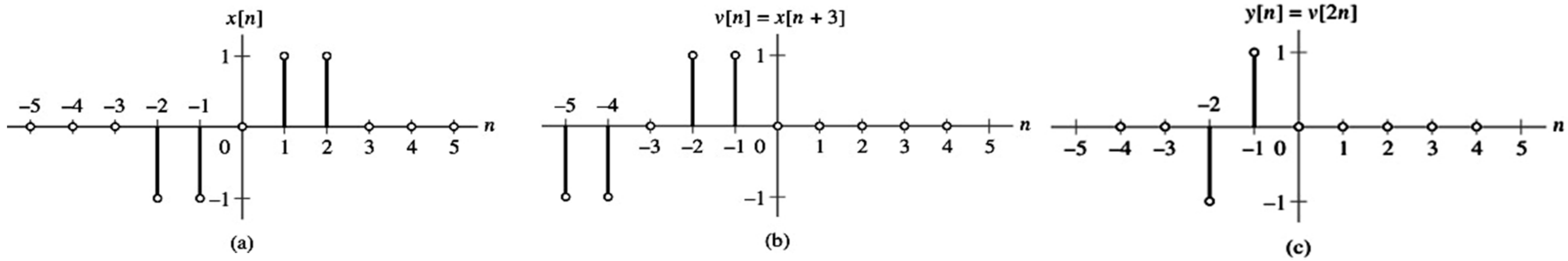


Figure 1.27
The proper order of applying the operations of time scaling and time shifting for the case of a discrete-time signal. (a) Discrete-time signal $x[n]$, antisymmetric about the origin. (b) Intermediate signal $v(n)$ obtained by shifting $x[n]$ to the left by 3 samples. (c) Discrete-time signal $y[n]$ resulting from the compression of $v[n]$ by a factor of 2, as a result of which two samples of the original $x[n]$, located at $n = -2, +2$, are lost.

Elementary Signals

- 1. **Exponential Signals** $x(t) = Be^{at}$ $x[n] = Br^{n}$
  - B and a can be real or complex parameters
  - Decaying exponential, if $a<0$; growing exponential, if a>0
  - Decaying exponential, if $0<r<1$; growing exponential, if $r>1$

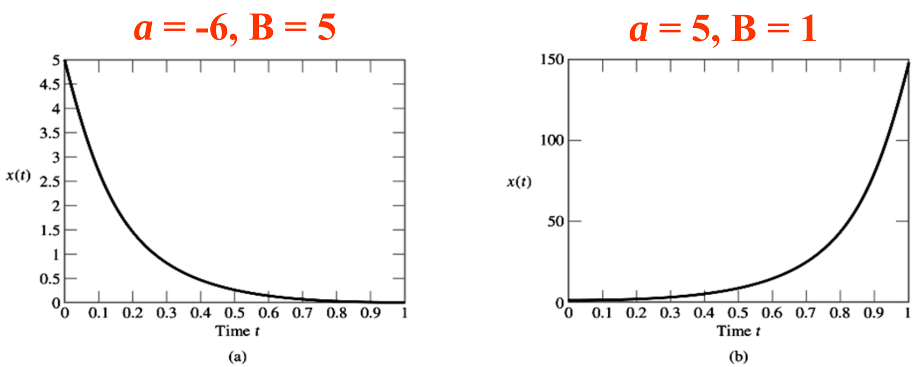


Figure 1.28 (a) Decaying exponential form of continuous-time signal. (b) Growing exponential form of continuous-time signal.

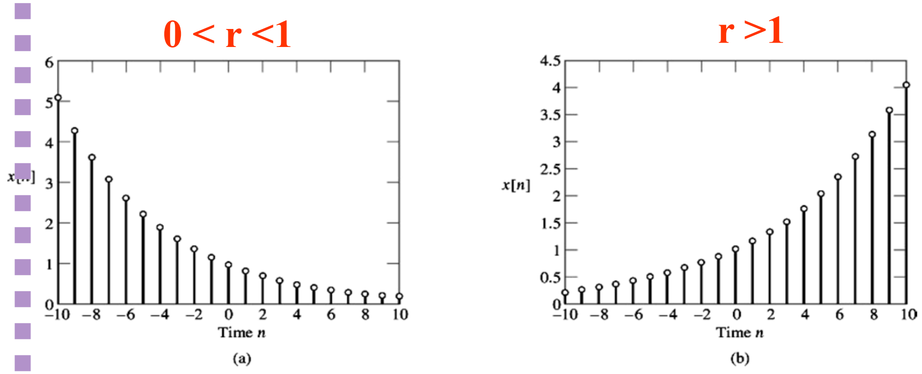


Figure 1.30 (a) Decaying exponential form of discrete-time signal. (b) Growing exponential form of discrete-time signal.

Elementary Signals

- 2. Sinusoidal Signals $x(t) = A\cos(\omega t + \phi)$ $x[n] = A\cos(\Omega n + \phi)$
  - Periodic continuous-time sinusoidal signals with period $T = 2\pi/\omega$
  - Discrete-time sinusoidal signals may or may not be periodic
    - Periodic if $\Omega = \dfrac{2\pi m}{N}$ radians/cycle, integer m, N

$$
\begin{aligned}
x(t+T) &= A\cos(\omega(t+T)+\phi) \\
&= A\cos(\omega t + \omega T + \phi) \\
&= A\cos(\omega t + 2\pi + \phi) \\
&= A\cos(\omega t + \phi) \\
&= x(t)
\end{aligned}
$$

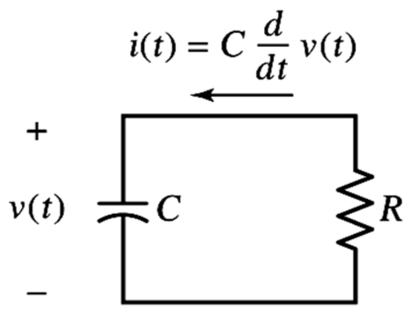
$$x[n+N] = A\cos(\Omega n + \Omega N + \phi)$$

$\Rightarrow$ $\Omega N = 2\pi m$ **or**

$$\Omega = \frac{2\pi m}{N} \quad \text{radians/cycle,} \quad \text{integer} \quad m, N$$

Physical Examples

Ex.1 – exponential signals



KVL Eq.:

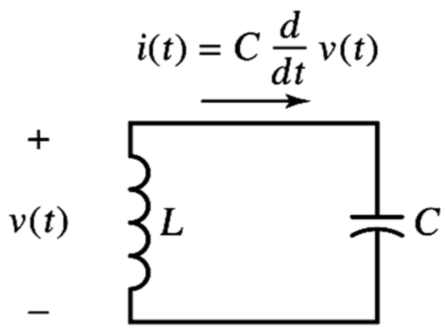
$$RC\frac{d}{dt}v(t) + v(t) = 0 \quad (1.32)$$

$$\Rightarrow v(t) = V_0 e^{-t/(RC)} \quad (1.33)$$

V_0: initial voltage across C
RC = Time constant,

Figure 1.29 Lossy capacitor, with the loss represented by shunt resistance R.

Ex.2 – sinusoidal signals



Circuit Eq.:

$$LC\frac{d^2}{dt^2}v(t) + v(t) = 0 \quad (1.36)$$

$$\Rightarrow v(t) = V_0\cos(\omega_0 t), \quad t \geq 0 \quad (1.37)$$

where

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (1.38)$$

Natural angular frequency of oscillation of the circuit

Figure 1.32 Parallel LC circuit, assuming that the inductor L and capacitor C are both ideal.

Example 1.7 Discrete-Time Sinusoidal Signals

- A pair of sinusoidal signals with a common angular frequency is defined by $x_1[n] = \sin[5\pi n],\ x_2[n] = \sqrt{3}\cos[5\pi n]$
  - (a) Both signals are periodic. Find their common fundamental period.
  - (b) Express the composite sinusoidal signal $y[n]=x_1[n]+x_2[n]$ in the form $y[n] = A\cos(\Omega n + \phi)$
- Sol.
  - (a) $\Omega = 5\pi$ radians/cycle ⇒ $N = \frac{2\pi m}{\Omega} = \frac{2\pi m}{5\pi} = \frac{2m}{5}$

    ⇒ This can be only for m = 5, 10, 15, …, which results in N = 2, 4, 6, …
  - (b) $A\cos(\Omega n + \phi) = A\cos(\Omega n)\cos(\phi) - A\sin(\Omega n)\sin(\phi)$

    $A\sin(\phi) = -1 \quad \text{and} \quad A\cos(\phi) = \sqrt{3}$

Solve ϕ and A

$$\tan(\phi) = \frac{\sin(\phi)}{\cos(\phi)} = \frac{\text{amplitude of } x_1[n]}{\text{amplitude of } x_2[n]} = \frac{-1}{\sqrt{3}}$$

⇒ $\phi = -\pi/6$

$$A\sin(\phi) = -1$$

$$A = \frac{-1}{\sin(-\pi/6)} = 2$$

Hence, we have

$$y[n] = 2\cos\left(5\pi n - \frac{\pi}{6}\right)$$

Euler's Identity

- 3. Relation Between Sinusoidal and Complex Exponential Signals
  - Euler's identity: $e^{j\theta} = \cos\theta + j\sin\theta$

$$\begin{aligned} &Be^{j\omega t} \\ &= Ae^{j\phi}e^{j\omega t} \\ &= Ae^{j(\phi+\omega t)} \\ &= A\cos(\omega t+\phi) + jA\sin(\omega t+\phi) \end{aligned}$$

$$A\cos(\omega t+\phi) = \mathrm{Re}\{Be^{j\omega t}\}$$

$$A\sin(\omega t+\phi) = \mathrm{Im}\{Be^{j\omega t}\}$$

$$A\sin(\Omega n+\phi) = \mathrm{Im}\{Be^{j\Omega n}\}$$

$$A\cos(\Omega n+\phi) = \mathrm{Re}\{Be^{j\Omega n}\}$$

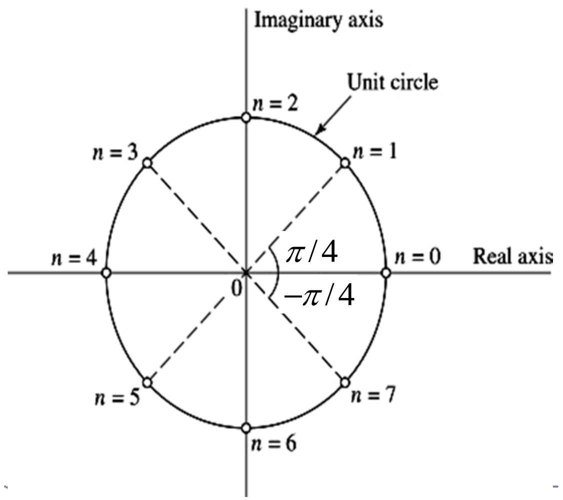


Figure 1.34 Complex plane, showing eight points uniformly distributed on the unit circle. The projection of the points on the real axis is cos(nπ/4), while the projection on the imaginary axis is sin(nπ/4); n=0,1, …, 7.

Elementary Signals

- 4. Exponential Damped Sinusoidal Signals
  - Continuous-time case $x(t) = Ae^{-\alpha t}\sin(\omega t + \phi), \quad \alpha > 0$
  - Discrete-time case $x[n] = Br^{n}\sin[\Omega n + \phi], \quad 0 < |r| < 1$

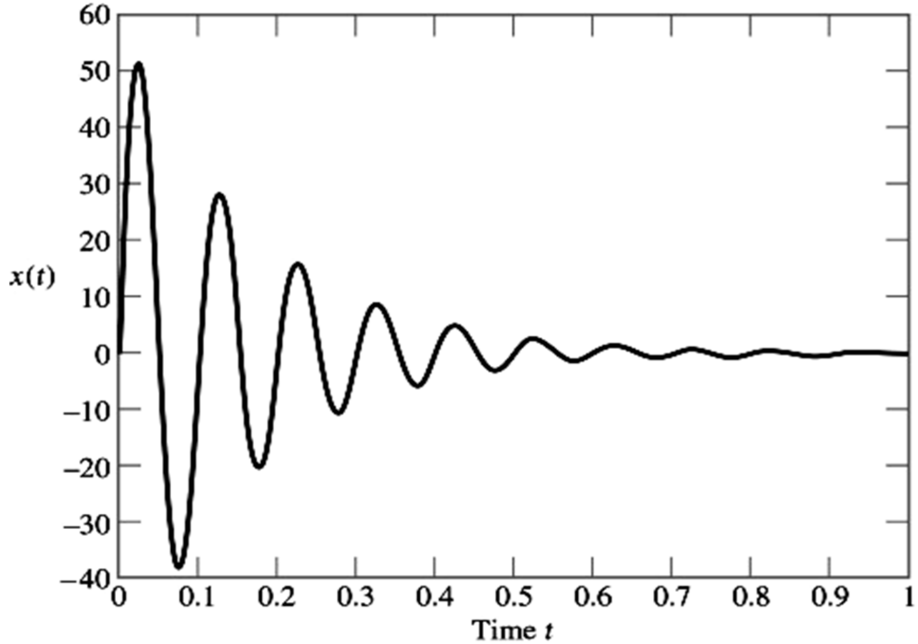


Figure 1.35 (p. 41)
Exponentially damped sinusoidal signal $Ae^{-\alpha t}\sin(\omega t)$, with A=60 and α=6.

Elementary Signals

- 5. Step Function
  - Discrete-time case

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

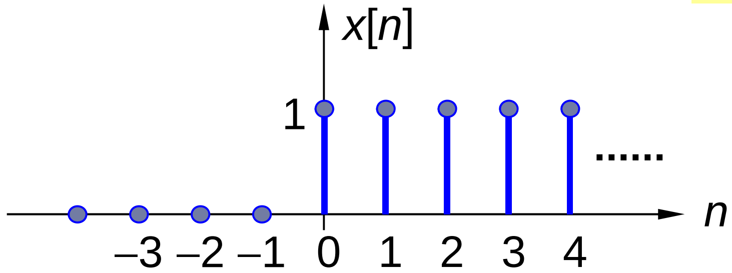


Figure 1.37
Discrete-time version of step function of unit amplitude.

  - Continuous-time case

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

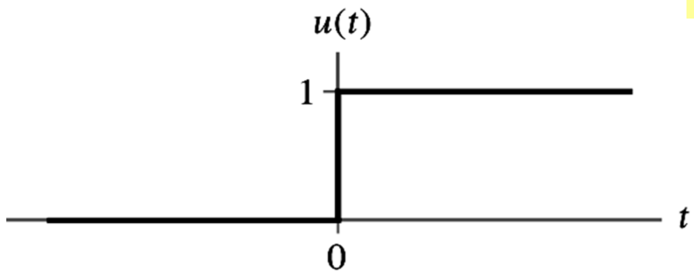
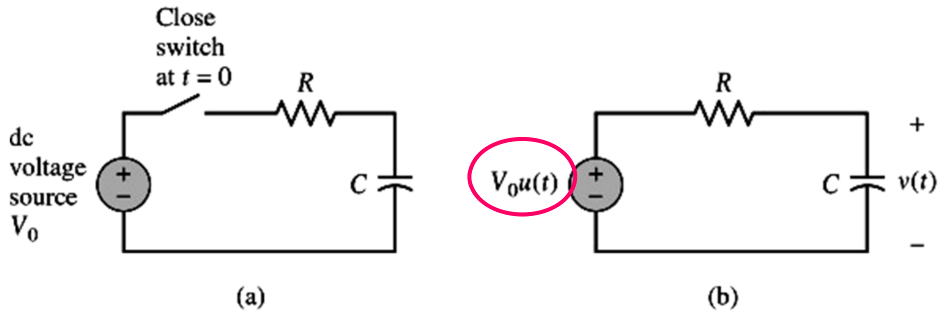


Figure 1.38
Continuous-time version of the unit-step function of unit amplitude.

Unit-Step Function Application

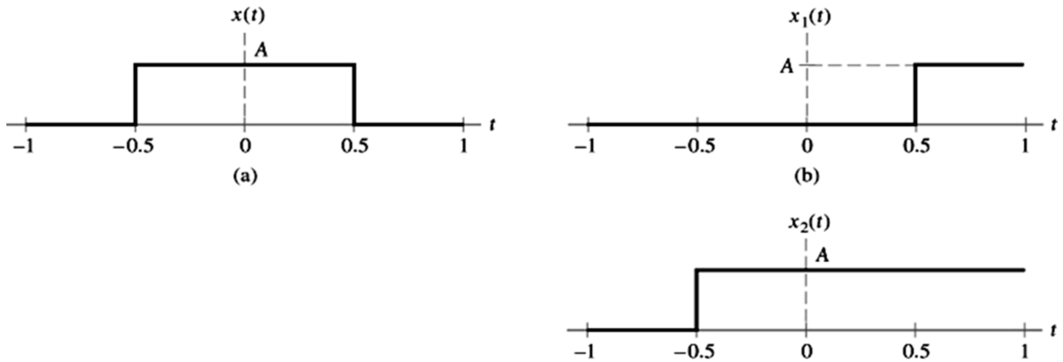
- $u(t)$ is a particularly simple signal to apply



$$v(t) = V_0\left(1 - e^{-t/(RC)}\right)u(t)$$

1. Initial value: $v(0) = 0$
2. Final value: $v(\infty) = V_0$

- $u(t)$ can be used to construct other discontinuous waveform, e.g. *rectangular pulse*



$$x(t) = Au\left(t + \frac{1}{2}\right) - Au\left(t - \frac{1}{2}\right)$$

$$x(t) = \begin{cases} A, & 0 \le |t| < 0.5 \\ 0, & |t| > 0.5 \end{cases}$$

Elementary Signals

- 6. Impulse Function
  - Discrete-time case $\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$

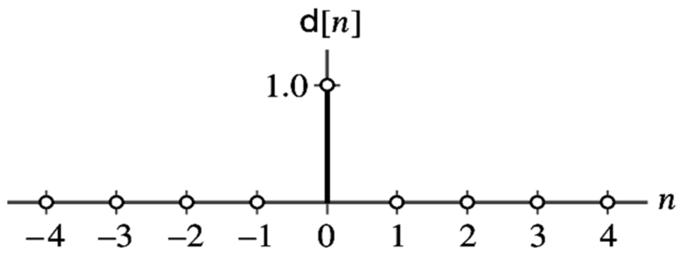


Figure 1.41
Discrete-time form of impulse.

  - Continuous-time case $\delta(t) = 0 \quad \text{for} \quad t \neq 0$ and $\int_{-\infty}^{\infty} \delta(t)dt = 1$

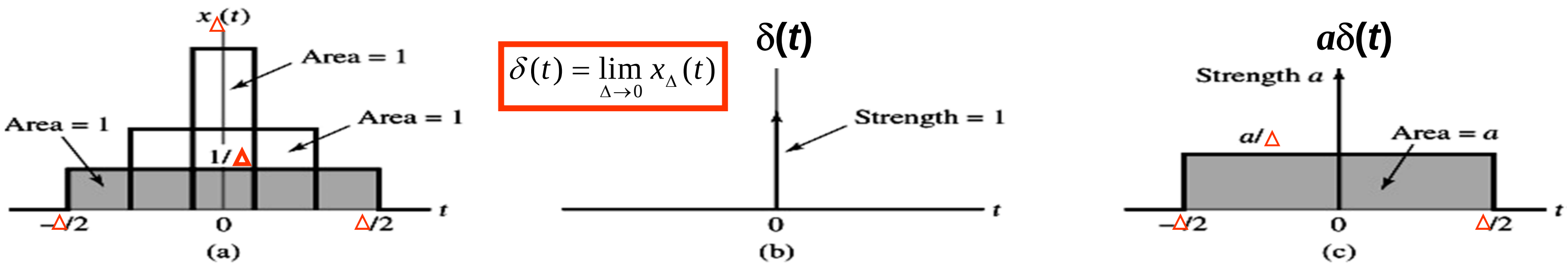


Figure 1.42
(a) Evolution of a rectangular pulse of unit area into an impulse of unit strength.
(b) Graphical symbol for unit impulse. (c) Representation of an impulse of strength *a*.

Impulse Function

- AKA Dirac delta function
  - $\delta(t)$ is zero everywhere except at the origin
  - The total area under the impulse $\delta(t)$ (or unit impulse), called the strength, is unity
- Mathematical relation between impulse and rectangular functions:

$$\delta(t) = \lim_{\Delta \to 0} x_\Delta(t)$$

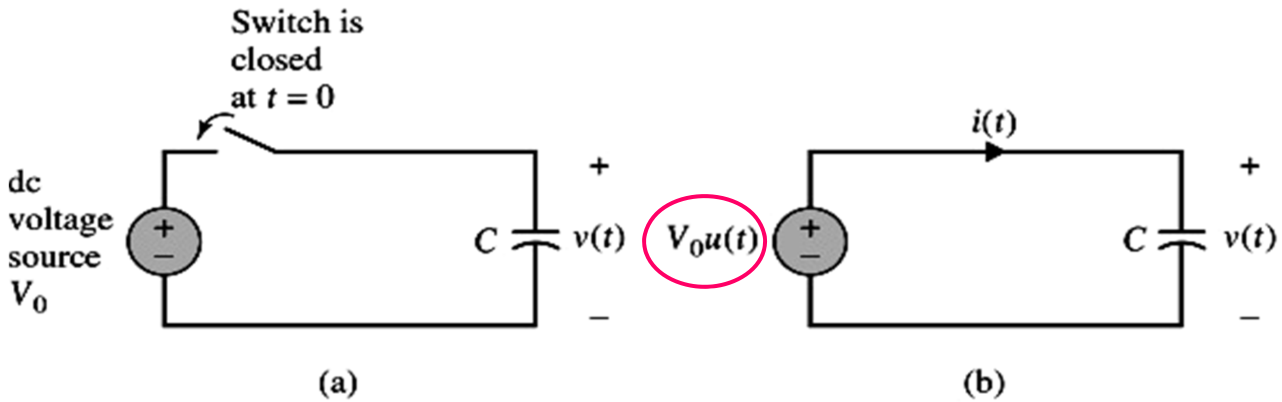
1. $x_\Delta(t)$: even function of t, Δ = duration.
2. $x_\Delta(t)$: Unit area.

- $\delta(t)$ is the derivative of $u(t)$; $u(t)$ is the integral of $\delta(t)$:

$$\delta(t) = \frac{d}{dt} u(t) \quad (1.62)$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau \quad (1.63)$$

RC Circuit Example (conti.)



1. Voltage across the capacitor:

$$v(t) = V_0 u(t)$$

2. Current flowing through capacitor:

$$i(t) = C\frac{dv(t)}{dt} \Rightarrow i(t) = CV_0\frac{du(t)}{dt} = CV_0\delta(t)$$

Properties of δ(t)

- Even function $\delta(-t) = \delta(t)$
- Shifting property $\int_{-\infty}^{\infty} x(t)\delta(t - t_0)dt = x(t_0)$ — Sampling at t_0 — If $x(t)$ is continuous at t_0
- Time-scaling property $\delta(at) = \frac{1}{a}\delta(t), \quad a > 0$

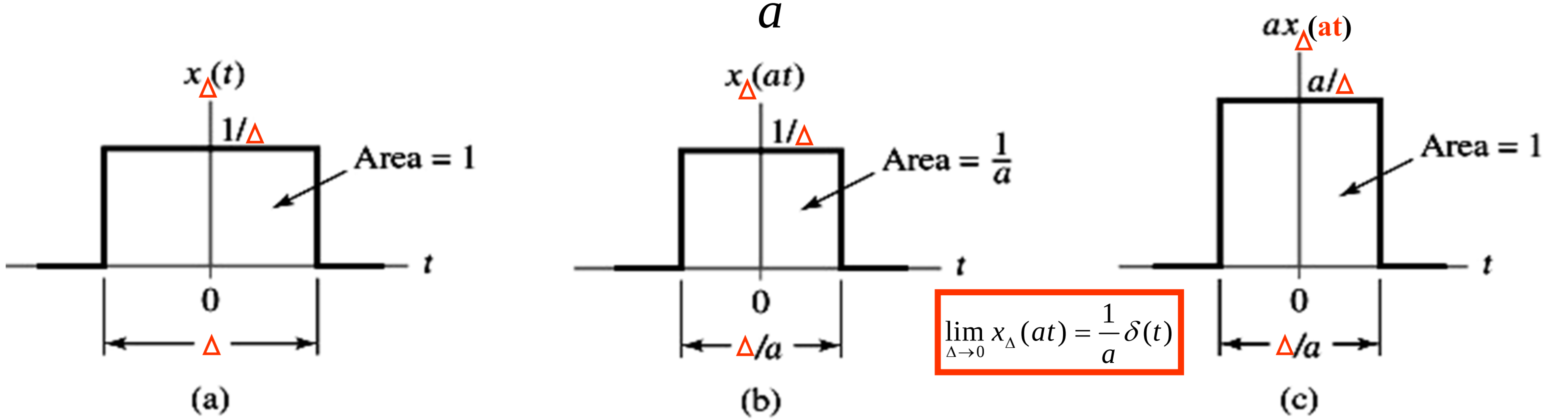


Figure 1.44 Steps involved in proving the time-scaling property of the unit impulse. (a) Rectangular pulse $x_\Delta(t)$ of amplitude 1/Δ and duration Δ, symmetric about the origin. (b) Pulse $x_\Delta(t)$ compressed by factor *a*. (c) Amplitude scaling of the compressed pulse, restoring it to unit area.

Elementary Signals

7. Derivation of the Impulse

- Doublet $\delta^{(1)}(t)$: the first derivative of $\delta(t)$
  - Recall Example 1.8, the rectangular pulse is $x(t) = Au\left(t+\frac{1}{2}\right) - Au\left(t-\frac{1}{2}\right)$

  ➔Unit rectangular pulse is equal to $\frac{1}{\Delta}(u(t+\frac{\Delta}{2}) - u(t-\frac{\Delta}{2}))$

$$\delta^{(1)}(t) = \lim_{\Delta \to 0} \frac{1}{\Delta}(\delta(t+\Delta/2) - \delta(t-\Delta/2))$$

- Fundamental property of the doublet

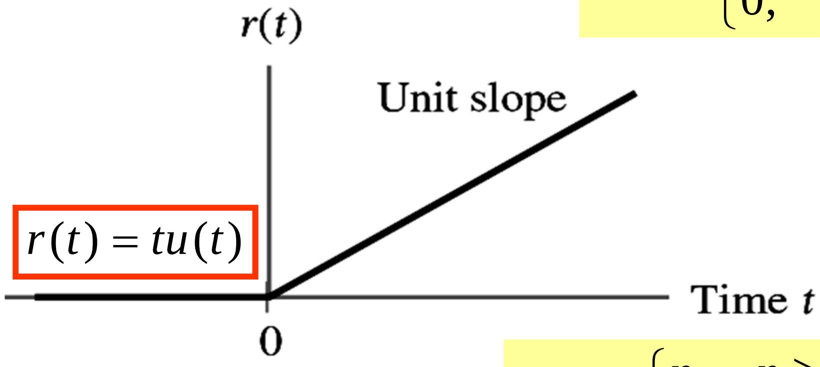
$$\int_{-\infty}^{\infty} f(t)\delta^{(1)}(t-t_0)dt = \frac{d}{dt} f(t)\Big|_{t=t_0} \qquad \int_{-\infty}^{\infty} \delta^{(1)}(t)dt = 0$$

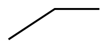
- Second derivative of impulse

$$\frac{\partial^2}{\partial t^2}\delta(t) = \frac{d}{dt}\delta^{(1)}(t) = \lim_{\Delta \to 0} \frac{\delta^{(1)}(t+\Delta/2) - \delta^{(1)}(t-\Delta/2)}{\Delta}$$

Elementary Functions

- 8. Ramp Function
  - Continuous-time case $r(t)=\begin{cases} t, & t \ge 0 \\ 0, & t < 0 \end{cases}$





The integral of $u(t)$

Figure 1.46
Ramp function of unit slope.

  - Discrete-time case $r[n]=\begin{cases} n, & n \ge 0 \\ 0, & n < 0 \end{cases}$

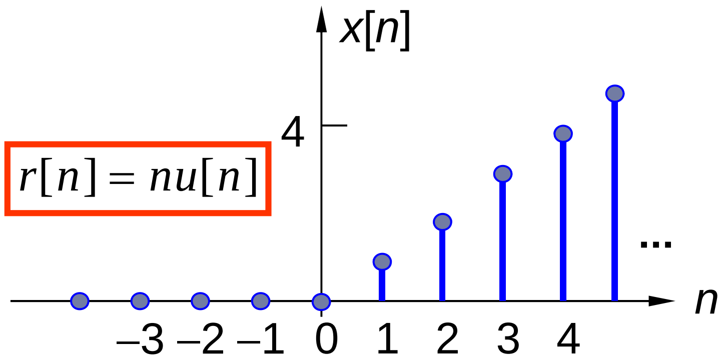
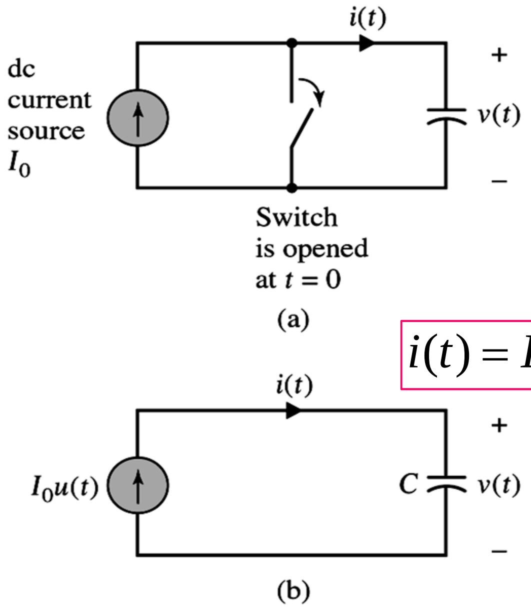


Figure 1.47
Discrete-time version of the ramp function.

Example 1.11 Parallel Circuit

Consider the parallel circuit of Fig. 1-48 (a) involving a dc current source I_0 and an initially uncharged capacitor C. The switch across the capacitor is suddenly opened at time t = 0. Determine the current *i(t)* flowing through the capacitor and the voltage *v(t)* across it for t ≥ 0.



$$i(t) = I_0u(t)$$

Figure 1.48(b) Equivalent circuit replacing the action of opening the switch with *u(t)*.

Capacitor voltage: $v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} I_0u(\tau)d\tau$$

$$= \begin{cases} 0 & \text{for } t < 0 \\ \frac{I_0}{C}t & \text{for } t > 1 \end{cases}$$

$$= \frac{I_0}{C}tu(t)$$

$$= \frac{I_0}{C}r(t)$$

A ramp function with slope I_0/C

Outline

- What is a signal?
- What is a system?
- Overview of specific systems
- Classification of signals
- Basic operations on signals
- Elementary signals
- Systems viewed as interconnections of operations
- Properties of systems
- Noises
- Theme example
- Exploring concepts with MATLAB

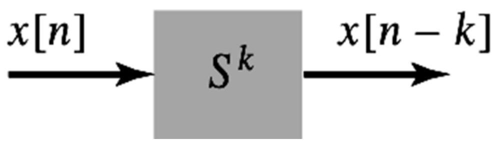
Systems Viewed as Interconnection of Operations

- A system is an interconnection of operations that transforms an input signal into an output signal
  - Let the operator $H\{\cdot\}$ denote the overall action of a system

(a) (b)

$$y(t) = H\{x(t)\} \qquad y[n] = H\{x[n]\}$$

  - Example: Discrete-time shift operator S^k:



Shifts the input by *k* time units

Example 1.12 Moving-Average System

Consider a discrete-time system whose output signal $y[n]$ is the average of three most recent values of the input signal $x[n]$, i.e. $y[n]=\frac{1}{3}(x[n]+x[n-1]+x[n-2])$ Formulate the operator H for this system; hence, develop a block diagram representation for it.

Sol. 1. Overall operator H for the moving-average system: $H=\frac{1}{3}(1+S+S^2)$

2.

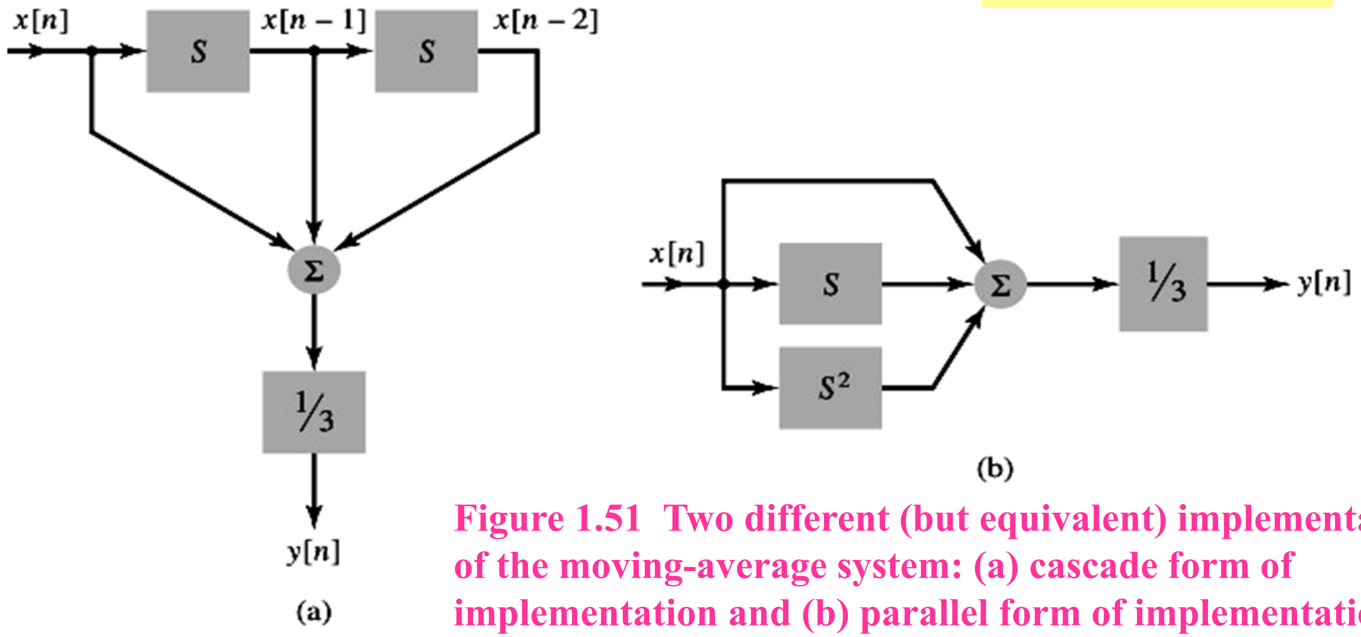


Figure 1.51 Two different (but equivalent) implementations of the moving-average system: (a) cascade form of implementation and (b) parallel form of implementation.

Properties of Systems

- 1. Stability
  - A system is said to be *bounded-input, bounded-output (BIBO) stable* iff every bounded input results in a bounded output.
  - The operator H is *BIBO stable* if

$$|y(t)| \le M_y < \infty \quad \forall t, \text{ whenever } |x(t)| \le M_x < \infty \quad \forall t.$$

  - Example 1.13
    - Finite moving-average system is BIBO stable

$$|y[n]| = \frac{1}{3}|x[n] + x[n-1] + x[n-2]|$$
$$\le \frac{1}{3}\left(|x[n]| + |x[n-1]| + |x[n-2]|\right)$$
$$\le \frac{1}{3}\left(M_x + M_x + M_x\right)$$
$$= M_x$$

Properties of Systems

- 2. Memory
  - A system is said to possess *memory* if its output signal depends on past or future values of the input signal.
    - Inductor $i(t)=\frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau$ Depends on the infinite past voltage
    - Moving-average system $y[n]=\frac{1}{3}(x[n]+x[n-1]+x[n-2])$

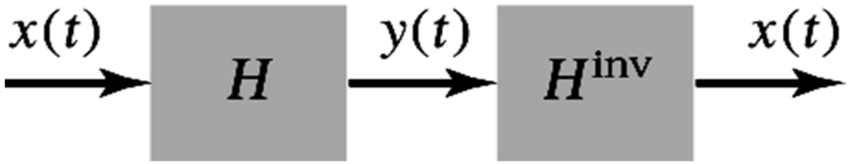
      Depends on two past values of $x[n]$
  - A system is said to possess *memoryless* if its output signal depends only on the present values of the input signal.
    - Resistor $i(t)=\frac{1}{R}v(t)$
    - A square-law system $y[n]=x^2[n]$

Properties of Systems

- 3. Causality
  - Causality is required for a system to be capable of operating in real time.
  - A system is said to be *causal* if its output signal depends only on the present or past values of the input signal.
  - A system is said to be *noncausal* if its output signal depends on one or more future values of the input signal.
  - For example,
    - Causal moving-average system, $y[n]=\frac{1}{3}(x[n]+x[n-1]+x[n-2])$
    - Noncausal moving-average system, $y[n]=\frac{1}{3}(x[n+1]+x[n]+x[n-1])$

Properties of Systems

- 4. Invertibility
  - A system is said to be *invertible* if the input of the system can be recovered from the output.



$$H^{inv}\{y(t)\} = H^{inv}\{H\{x(t)\}\} = H^{inv}H\{x(t)\}$$

$$\Rightarrow \quad \boxed{H^{inv}H = I}$$

Condition for an invertible system

    - H^{inv}: inverse operator; I: identity operator
  - A one-to-one mapping between input and output signals for a system is invertible
    - Distinct inputs applied to the system produce distinct outputs.
  - The inverse of the communication channel is aka the equalizer

Invertible and Noninvertible Systems Example

Example 1.15 – Inverse of System

Consider the time-shift system described by the input-output relation

$y(t) = x(t - t_0) = S^{t_0}\{x(t)\}$, **where the operator S^{t_0} represents a time shift of t_0 seconds. Find the inverse of this system.**

$$S^{-t_0}\{y(t)\} = S^{-t_0}\{S^{t_0}\{x(t)\}\} = S^{-t_0}S^{t_0}\{x(t)\}$$

$$S^{-t_0}S^{t_0} = I$$

Example 1.16 – Non-Invertible System

Show that a square-law system described by the input-output relation

$y(t) = x^2(t)$ **is not invertible.**

Since the distinct inputs $x(t)$ and $-x(t)$ produce the same output $y(t)$. Not 1-1 mapping. Accordingly, the square-law system is not invertible.

Properties of Systems

- 5. Time invariance
  - A system is said to be *time invariant* if a time delay (or time advance) of the input signal leads to an identical time shift in the output signal.
    - A time-invariant system responds identically no matter when the input signal is applied.

(a) (b)

$$\begin{aligned} y_2(t) &= H\{x_1(t-t_0)\} \\ &= H\{S^{t_0}\{x_1(t)\}\} \\ &= HS^{t_0}\{x_1(t)\} \end{aligned}$$

$$\begin{aligned} y_1(t-t_0) &= S^{t_0}\{y_1(t)\} \\ &= S^{t_0}\{H\{x_1(t)\}\} \\ &= S^{t_0}H\{x_1(t)\} \end{aligned}$$

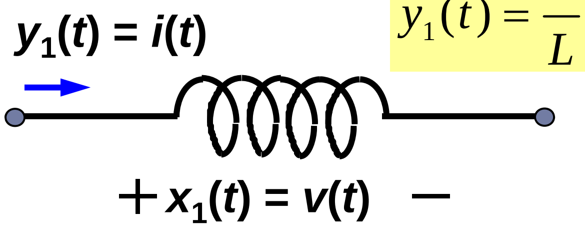
Condition for time-invariant system: $\Rightarrow$ $HS^{t_0} = S^{t_0}H$

H and S^{t_0} must be communicated with each other

Example 1.17 vs. Example 1.18

- Ex. 1.17 Inductor

$$y_1(t) = \frac{1}{L}\int_{-\infty}^{t} x_1(\tau)\,d\tau$$



$$y_2(t) = \frac{1}{L}\int_{-\infty}^{t} x_1(\tau - t_0)\,d\tau$$

Changing variables: $\tau' = \tau - t_0$

$$\Rightarrow\quad y_2(t) = \frac{1}{L}\int_{-\infty}^{t-t_0} x_1(\tau')\,d\tau'$$

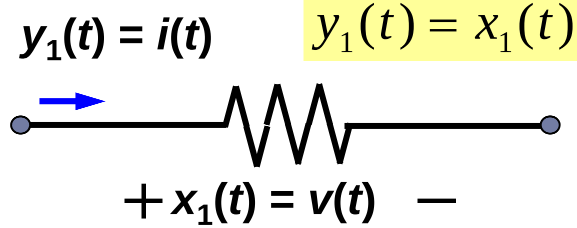
$$\parallel$$

$$y_1(t - t_0) = \frac{1}{L}\int_{-\infty}^{t-t_0} x_1(\tau)\,d\tau$$

Inductor is time invariant.

- Ex. 1.18 Thermistor

$$y_1(t) = x_1(t)/R(t)$$



$$y_2(t) = \frac{x_1(t - t_0)}{R(t)}$$

$$y_1(t - t_0) = \frac{x_1(t - t_0)}{R(t - t_0)}$$

Since $R(t) \neq R(t - t_0)$

$\Rightarrow$ $y_1(t - t_0) \neq y_2(t) \quad \text{for} \quad t_0 \neq 0$

$\Rightarrow$ ***Thermistor is time variant***

Properties of Systems

- 6. Linearity
  - Superposition property

$$H\{x_1(t)\} + H\{x_2(t)\} = y_1(t) + y_2(t) = H\{x_1(t) + x_2(t)\}$$

  - Homogeneity property

$$aH\{x_1(t)\} = ay_1(t) = H\{ax_1(t)\}, \quad a = \text{constant factor}$$

  - A system is said to be *linear* if it satisfies the superposition and homogeneity properties
    - If $x(t) = \sum_{i=1}^{N} a_i x_i(t)$ (1.86) $x_1(t), x_2(t), \ldots, x_N(t) \equiv$ input signal; $a_1, a_2, \ldots, a_N \equiv$ Corresponding weighted factor
    - then $y(t) = H\{x(t)\} = H\{\sum_{i=1}^{N} a_i x_i(t)\}$ $\xrightarrow{\text{linear}}$ $y(t) = \sum_{i=1}^{N} a_i y_i(t)$

Linear Systems

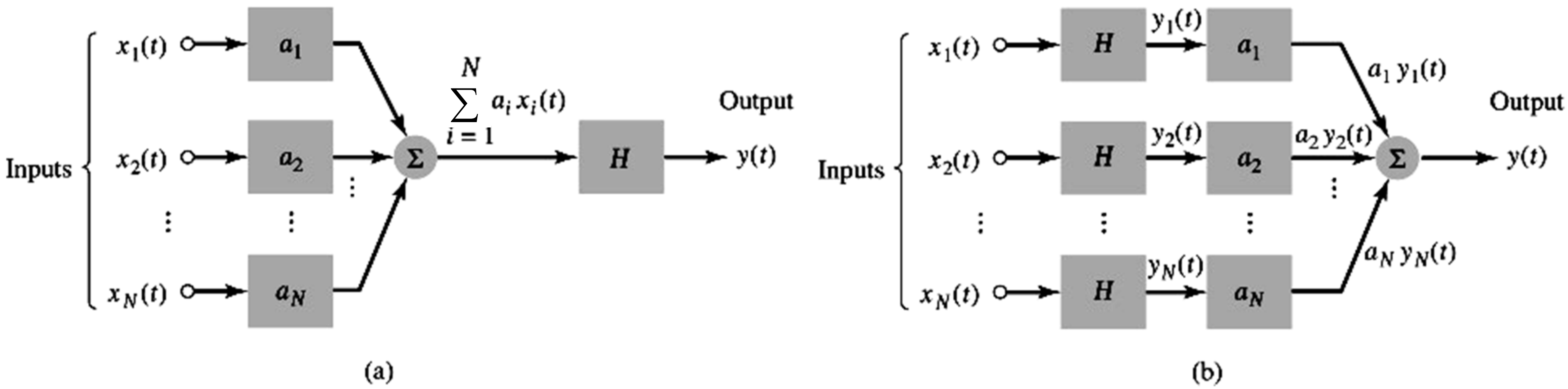


Figure 1.56 The linearity property of a system. (a) The combined operation of amplitude scaling and summation precedes the operator *H* for multiple inputs. (b) The operator *H* precedes amplitude scaling for each input; the resulting outputs are summed to produce the overall output *y*(*t*). If these two configurations produce the same output *y*(*t*), the operator *H* is linear.

Outline