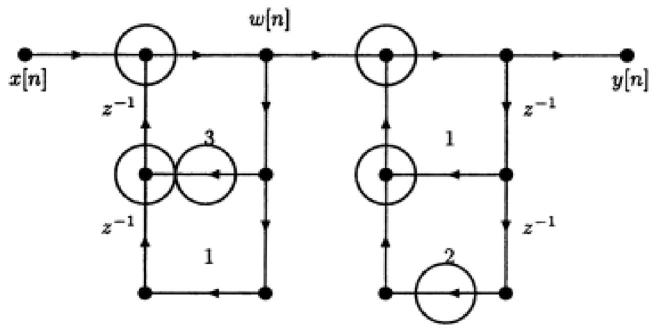
DSP HW5 Sol

6.3.

$$H(z) = \frac{2 + \frac{1}{4}z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}$$

System (d) is recognizable as a transposed direct form II implementation of $H(z)$.

6.5. The flow graph for this system is drawn below.



(a)

$$w[n] = x[n] + 3w[n-1] + w[n-2]$$
$$y[n] = w[n] + y[n-1] + 2y[n-2]$$

(b)

$$W(z) = X(z) + 3z^{-1}W(z) + z^{-2}W(z)$$
$$Y(z) = W(z) + z^{-1}Y(z) + 2z^{-2}Y(z)$$

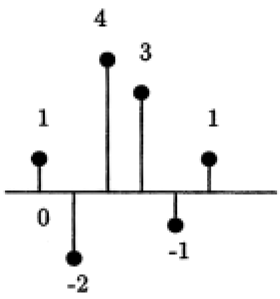
So

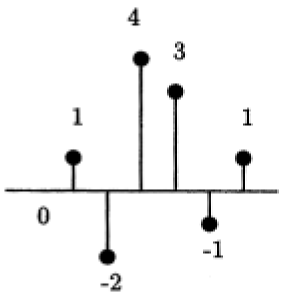
$$\begin{aligned}\frac{Y(z)}{X(z)} &= H(z) \\ &= \frac{1}{(1 - z^{-1} - 2z^{-2})(1 - 3z^{-1} - z^{-2})} \\ &= \frac{1}{1 - 4z^{-1} + 7z^{-3} + 2z^{-4}}.\end{aligned}$$

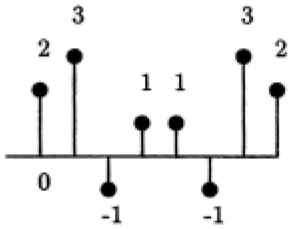
(c) Adds and multiplies are circled above: 4 real adds and 2 real multiplies per output point.

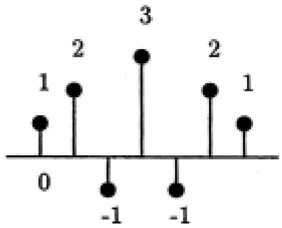
(d) It is not possible to reduce the number of storage registers. Note that implementing $H(z)$ above in the canonical direct form II (minimum storage registers) also requires 4 registers.

6.6. The impulse responses of each system are shown below.

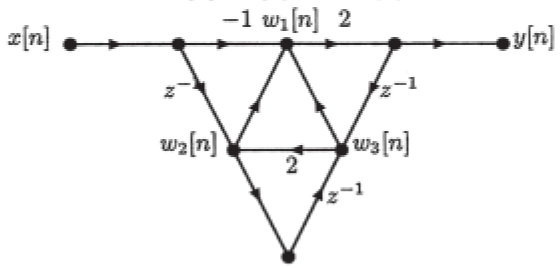
(a)


(b)


(c)


(d)


6.12. We define the intermediate variables $w_1[n]$, $w_2[n]$ and $w_3[n]$ as follows:



We thus have the following relationships:

$$
\begin{aligned}
w_1[n] &= -x[n] + w_2[n] + w_3[n] \\
w_2[n] &= x[n-1] + 2w_3[n] \\
w_3[n] &= w_2[n-1] + y[n-1] \\
y[n] &= 2w_1[n].
\end{aligned}
$$

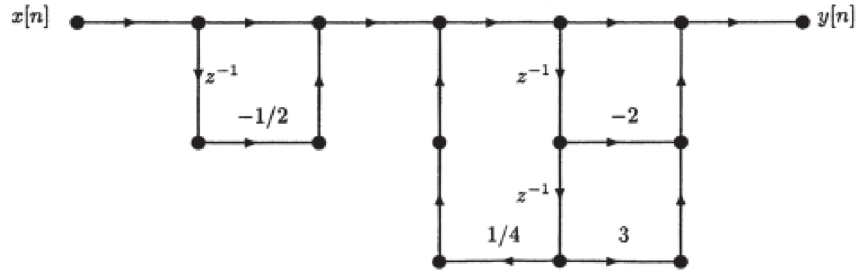
$Z-$transforming the above equations and rearranging and grouping terms, we get:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-2 + 6z^{-1} + 2z^{-2}}{1 - 8z^{-1}}.$$

Taking the inverse $Z-$transform, we get the following difference equation:

$$y[n] - 8y[n-1] = -2x[n] + 6x[n-1] + 2x[n-2].$$

6.16. (a) We just reverse the arrows and reverse the role of the input and the output, we get:



(b) The original system is the cascade of two transposed direct form II structures, therefore the system function is given by:

$$H(z) = (\frac{1 - 2z^{-1} + 3z^{-2}}{1 - \frac{1}{4}z^{-2}})(1 - \frac{1}{2}z^{-1}).$$

The transposed graph, on the other hand, is the cascade of two direct form II structures, therefore the system function is given by:

$$H(z) = (1 - \frac{1}{2}z^{-1})(\frac{1 - 2z^{-1} + 3z^{-2}}{1 - \frac{1}{4}z^{-2}}).$$

This confirms that both graphs have the same system function $H(z)$.

6.18. The flow graph is just a cascade of two transposed direct form II structures, the system function is thus given by:

$$H(z) = (\frac{1 + \frac{4}{3}z^{-1} - \frac{4}{3}z^{-2}}{1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}})(\frac{1}{1 - az^{-1}}).$$

Which can be rewritten as:

$$H(z) = \frac{(1 + 2z^{-1})(1 - \frac{2}{3}z^{-1})}{(1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2})(1 - az^{-1})}.$$

In order to implement this system function with a second-order direct form II signal flow graph, a pole-zero cancellation has to occur, this happens if $a = \frac{2}{3}$, $a = -2$ or $a = 0$. If $a = \frac{2}{3}$, the overall system function is:

$$H(z) = \frac{1 + 2z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}.$$

If $a = -2$, the overall system function is:

$$H(z) = \frac{1 - \frac{2}{3}z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}.$$

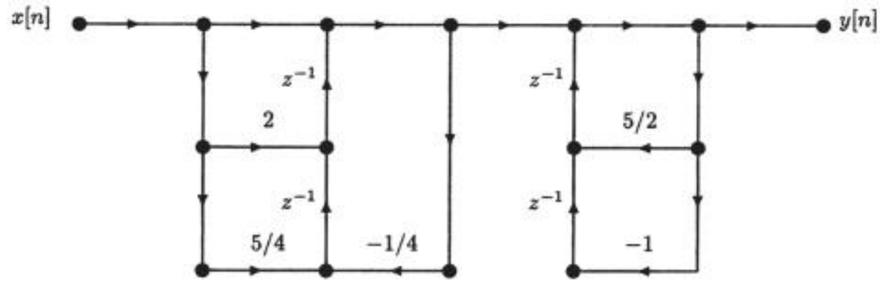
And finally if $a = 0$, the overall system function is:

$$H(z) = \frac{(1 + 2z^{-1})(1 - \frac{2}{3}z^{-1})}{1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}.$$

6.20. The transfer function can be rewritten as:

$$H(z) = \frac{(1 + 2z^{-1} + \frac{5}{4}z^{-2})}{(1 + \frac{1}{4}z^{-2})(1 - \frac{5}{2}z^{-1} + z^{-2})}$$

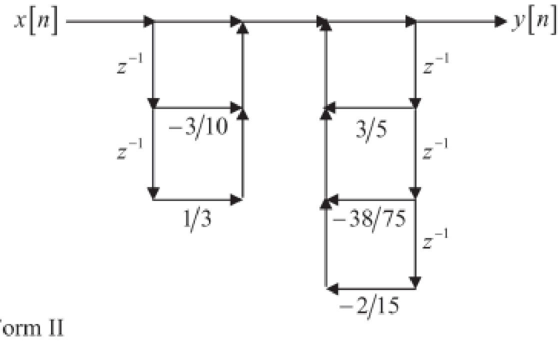
which can be implemented as the following cascade of second-order transposed direct form II sections:



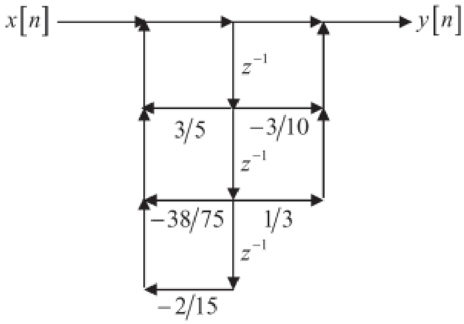
6.24.

$$H(z) = \frac{1 - \frac{3}{10}z^{-1} + \frac{1}{3}z^{-2}}{\left(1 - \frac{4}{5}z^{-1} + \frac{2}{3}z^{-2}\right)\left(1 + \frac{1}{5}z^{-1}\right)}$$
$$= \frac{1 - \frac{3}{10}z^{-1} + \frac{1}{3}z^{-2}}{1 - \frac{3}{5}z^{-1} + \frac{38}{75}z^{-2} + \frac{2}{15}z^{-3}}.$$

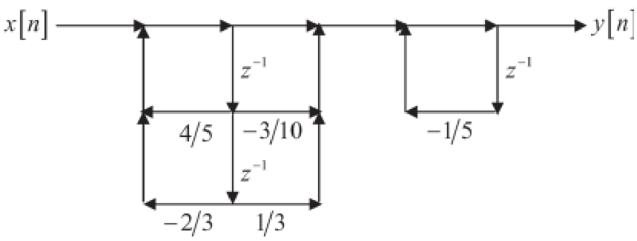
1. i) Direct Form I



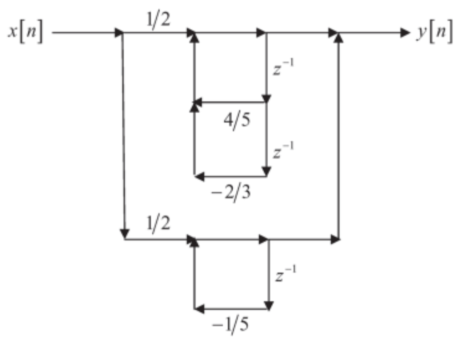
ii) Direct Form II



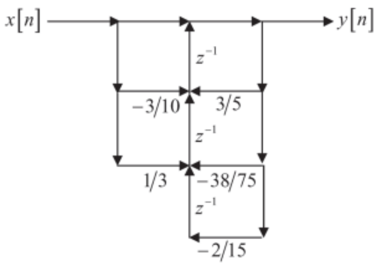
iii) Cascade Form



iv) Parallel Form



v) Transposed Direct Form II



2. Label the interior nodes of the transposed direct form II structure $v_0[n], v_1[n], v_2[n], v_3[n]$ counting from the top down. Then we have

$$
\begin{aligned}
y[n] &= v_0[n] \\
v_0[n] &= x[n] + v_1[n-1] \\
v_1[n] &= \tfrac{3}{5} y[n] - \tfrac{3}{10} x[n] + v_2[n-1] \\
v_2[n] &= -\tfrac{38}{75} y[n] + \tfrac{1}{3} x[n] + v_3[n-1] \\
v_3[n] &= -\tfrac{2}{15} y[n].
\end{aligned}
$$

Taking the z-transform of these equations gives

$$
\begin{aligned}
Y(z) &= V_0(z) \\
V_0(z) &= X(z) + z^{-1} V_1(z) \\
V_1(z) &= \tfrac{3}{5} Y(z) - \tfrac{3}{10} X(z) + z^{-1} V_2(z) \\
V_2(z) &= -\tfrac{38}{75} Y(z) + \tfrac{1}{3} X(z) + z^{-1} V_3(z) \\
V_3(z) &= -\tfrac{2}{15} Y(z).
\end{aligned}
$$

Substituting Eq. (5) into Eq. (4) gives

$$
\begin{aligned}
V_2(z) &= -\tfrac{38}{75} Y(z) + \tfrac{1}{3} X(z) - \tfrac{2}{15} z^{-1} Y(z) \\
&= -\left(\tfrac{38}{75} + \tfrac{2}{15} z^{-1}\right) Y(z) + \tfrac{1}{3} X(z).
\end{aligned}
$$

Substituting into Eq. (3) gives

$$
\begin{aligned}
V_1(z) &= \tfrac{3}{5} Y(z) - \tfrac{3}{10} X(z) + z^{-1}\left(-\left(\tfrac{38}{75} + \tfrac{2}{15} z^{-1}\right) Y(z) + \tfrac{1}{3} X(z)\right) \\
&= \left(\tfrac{3}{5} - \tfrac{38}{75} z^{-1} - \tfrac{2}{15} z^{-2}\right) Y(z) + \left(-\tfrac{3}{10} + \tfrac{1}{3} z^{-1}\right) X(z).
\end{aligned}
$$

Now substitute into Eq. (2):

$$
\begin{aligned}
V_0(z) &= X(z) + z^{-1}\left\{\left(\tfrac{3}{5} - \tfrac{38}{75} z^{-1} - \tfrac{2}{15} z^{-2}\right) Y(z) + \left(-\tfrac{3}{10} + \tfrac{1}{3} z^{-1}\right) X(z)\right\} \\
&= \left(1 - \tfrac{3}{10} z^{-1} + \tfrac{1}{3} z^{-2}\right) X(z) + \left(\tfrac{3}{5} z^{-1} - \tfrac{38}{75} z^{-2} - \tfrac{2}{15} z^{-3}\right) Y(z).
\end{aligned}
$$

Finally, substitute into Eq. (1):

$$
Y(z) = \left(1 - \tfrac{3}{10} z^{-1} + \tfrac{1}{3} z^{-2}\right) X(z) + \left(\tfrac{3}{5} z^{-1} - \tfrac{38}{75} z^{-2} - \tfrac{2}{15} z^{-3}\right) Y(z)
$$

$$
\left(1 - \tfrac{3}{5} z^{-1} + \tfrac{38}{75} z^{-2} + \tfrac{2}{15} z^{-3}\right) Y(z) = \left(1 - \tfrac{3}{10} z^{-1} + \tfrac{1}{3} z^{-2}\right) X(z)
$$

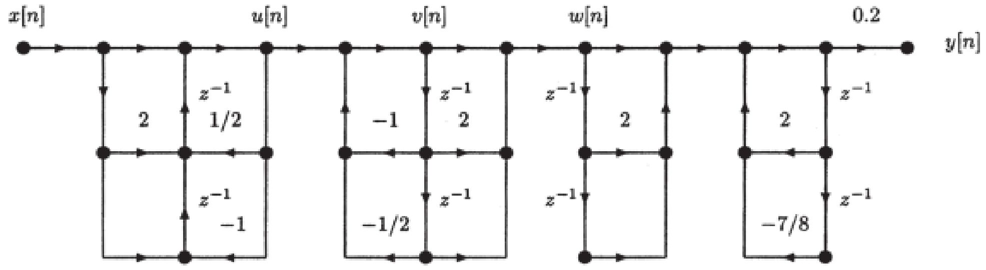
$$
H(z) = \frac{Y(z)}{X(z)} = \frac{\left(1 - \tfrac{3}{10} z^{-1} + \tfrac{1}{3} z^{-2}\right)}{\left(1 - \tfrac{3}{5} z^{-1} + \tfrac{38}{75} z^{-2} + \tfrac{2}{15} z^{-3}\right)}.
$$

This final expression is the correct system function.

6.29.

(a) We can rearrange $H(z)$ this way:

$$H(z) = \frac{(1+z^{-1})^2}{1-\frac{1}{2}z^{-1}+z^{-2}} \cdot \frac{(1+z^{-1})^2}{1+z^{-1}+\frac{1}{2}z^{-2}} \cdot (1+z^{-1})^2 \cdot \frac{1}{1-2z^{-1}+\frac{7}{8}z^{-2}} \cdot 0.2$$



The solution is not unique; the order of the denominator 2nd-order sections may be rearranged.

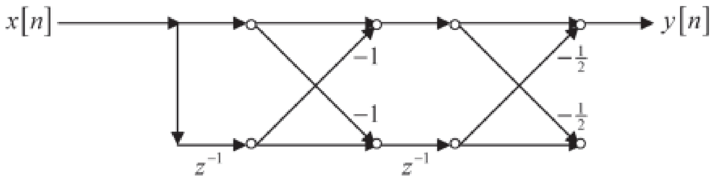
(b)

$$\begin{aligned} u[n] &= x[n] + 2x[n-1] + x[n-2] + \frac{1}{2}u[n-1] - u[n-2] \\ v[n] &= u[n] - v[n-1] - \frac{1}{2}v[n-2] \\ w[n] &= v[n] + 2v[n-1] + v[n-2] \\ y[n] &= w[n] + 2w[n-1] + w[n-2] + 2y[n-1] - \frac{7}{8}y[n-2]. \end{aligned}$$

6.31. (a) To determine $y[1]$, sum the gains of all paths with a single delay to the output. This gives

$$y[1] = 1 + (-1)(\tfrac{1}{2}) = \tfrac{1}{2}.$$

(b) The flow graph for the inverse filter will be a cascade of FIR stages with the *k*-coefficients in the reverse order.



(c) When the FIR lattice of part (b) is driven by an impulse, the response is seen to be

$$\begin{aligned} h_{FIR}[n] &= \delta[n] + (-1 + (-1)(-\tfrac{1}{2}))\delta[n-1] - \tfrac{1}{2}\delta[n-2] \\ &= \delta[n] - \tfrac{1}{2}\delta[n-1] - \tfrac{1}{2}\delta[n-2]. \end{aligned}$$

The transfer function is

$$H_{FIR}(z) = 1 - \tfrac{1}{2}z^{-1} - \tfrac{1}{2}z^{-2}.$$

This is the transfer function for the inverse filter. The transfer function for the given lattice is then

$$H(z) = \frac{1}{1 - \frac{1}{2}z^{-1} - \frac{1}{2}z^{-2}}.$$

6.41. (a)

$$\begin{aligned} y_1[n] &= (1+r)x_1[n] + rx_2[n] \\ y_2[n] &= -rx_1[n] + (1-r)x_2[n]. \end{aligned}$$

(b)

$$\begin{aligned} y_1[n] &= (1+a)x_1[n] + dx_2[n] \quad (a = r = d) \\ y_2[n] &= (1+cd)x_2[n] + abx_1[n] \quad (c = d = -1). \end{aligned}$$

(c)

$$\begin{aligned} y_1[n] &= (1+e)x_1[n] + ex_2[n] \quad (e = r) \\ y_2[n] &= efx_1[n] + (1+ef)x_2[n] \quad (f = -1). \end{aligned}$$

(d) B and C preferred over A:

(i) coefficient quantization. If r is small, $1+r$ may not be precisely representable even in floating point. Also, network A has 4 multipliers that must be quantized, while B and C have only 1.

(ii) computational complexity. Networks B and C require fewer multiplications per output sample.