4.4. (a) Letting $T = 1/100$ gives

$$\begin{aligned} x[n] &= x_c(nT) \\ &= \sin\left(20\pi n \frac{1}{100}\right) + \cos\left(40\pi n \frac{1}{100}\right) \\ &= \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right) \end{aligned}$$

(b) No, another choice is $T = 11/100$:

$$\begin{aligned} x[n] &= x_c(nT) \\ &= \sin\left(20\pi n \frac{11}{100}\right) + \cos\left(40\pi n \frac{11}{100}\right) \\ &= \sin\left(\frac{11\pi n}{5}\right) + \cos\left(\frac{22\pi n}{5}\right) \\ &= \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right) \end{aligned}$$

4.7. The continuous-time signal contains an attenuated replica of the original signal with a delay of τ_d.

$$x_c(t) = s_c(t) + \alpha s_c(t - \tau_d)$$

(a) Taking the Fourier transform of the analog signal:

$$X_c(j\Omega) = S_c(j\Omega) \cdot (1 + \alpha e^{-j\tau_d \Omega})$$

Note that $X_c(j\Omega)$ is zero for $|\Omega| > \pi/T$. Sampling the continuous-time signal yields the discrete-time sequence, $x[n]$. The Fourier transform of the sequence is

$$\begin{aligned} X(e^{j\omega}) &= \frac{1}{T}\sum_{r=-\infty}^{\infty} S_c(\frac{j\omega}{T} + j\frac{2\pi r}{T}) \\ &+ \frac{\alpha}{T}\sum_{r=-\infty}^{\infty} S_c(\frac{j\omega}{T} + j\frac{2\pi r}{T}) e^{-j\tau_d(\frac{\omega}{T} + \frac{2\pi r}{T})} \end{aligned}$$

(b) The desired response:

$$H(j\Omega) = \begin{cases} 1 + \alpha e^{-j\tau_d \Omega}, & \text{for } |\Omega| \le \frac{\pi}{T} \\ 0, & \text{otherwise} \end{cases}$$

Using $\omega = \Omega T$, we obtain a discrete-time system which simulates the above response:

$$H(e^{j\omega}) = 1 + \alpha e^{-j\frac{\tau_d \omega}{T}}$$

(c) We need to take the inverse Fourier transform of the discrete-time impulse response of part (b).

$$\begin{aligned} h[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n}\, d\omega \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} (1 + \alpha e^{-j\frac{\tau_d \omega}{T}}) e^{j\omega n}\, d\omega \end{aligned}$$

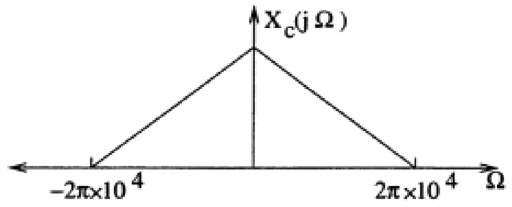
(i) Consider the case when $\tau_d = T$:

$$\begin{aligned} h[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} (e^{j\omega n} + \alpha e^{j\omega(n-1)})\, d\omega \\ &= \frac{\sin(\pi n)}{\pi n} + \frac{\alpha \sin[\pi(n-1)]}{\pi(n-1)} \\ &= \delta[n] + \alpha\delta[n-1] \end{aligned}$$

(ii) For $\tau_d = T/2$:

$$\begin{aligned} h[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} (e^{j\omega n} + \alpha e^{j\omega(n-\frac{1}{2})})\, d\omega \\ &= \frac{\sin(\pi n)}{\pi n} + \frac{\alpha \sin[\pi(n-\frac{1}{2})]}{\pi(n-\frac{1}{2})} \\ &= \delta[n] + \frac{\alpha \sin[\pi(n-\frac{1}{2})]}{\pi(n-\frac{1}{2})} \end{aligned}$$

4.8. A plot of $X_c(j\Omega)$ appears below.



(a) For $x_c(t)$ to be recoverable from $x[n]$, the transform of the discrete signal must have no aliasing. When sampling, the radian frequency is related to the analog frequency by

$$\omega = \Omega T.$$

No aliasing will occur if the sampling interval satisfies the Nyquist Criterion. Thus, for the band-limited signal, $x_c(t)$, we should select T as:

$$T \leq \frac{1}{2 \times 10^4}.$$

(b) Assuming that the system is linear and time-invariant, the convolution sum describes the input-output relationship.

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

We are given

$$\begin{aligned} y[n] &= T \sum_{k=-\infty}^{n} x[k] \\ &= T \sum_{k=-\infty}^{\infty} x[k]u[n-k] \end{aligned}$$

Hence, we may infer that the impulse response of the system

$$h[n] = T \cdot u[n].$$

(c) We use the expression for $y[n]$ as given and examine the limit

$$\begin{aligned} \lim_{n\to\infty} y[n] &= \lim_{n\to\infty} T \cdot \sum_{k=-\infty}^{n} x[k] \\ &= T \cdot \sum_{k=-\infty}^{\infty} x[k] \end{aligned}$$

Recall the analysis equation for the Fourier transform:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Hence,

$$\lim_{n\to\infty} y[n] = T \cdot X(e^{j\omega})|_{\omega=0}$$

(d) We use the result from part (c). Noting that

$$X(e^{j\omega}) = \frac{1}{T}\sum_{r=-\infty}^{\infty} X_c(\frac{j\omega}{T} + \frac{j2\pi r}{T}).$$

Thus, we have

$$T \cdot X(e^{j\omega})|_{\omega=0} = \sum_{r=-\infty}^{\infty} X_c(\frac{j2\pi r}{T})$$

From the given information, we seek a value of T such that:

$$\begin{aligned}\sum_{r=-\infty}^{\infty} X_c(\frac{j2\pi r}{T}) &= \int_{-\infty}^{\infty} x_c(t)\,dt \\ &= X_c(j\Omega)|_{\Omega=0}\end{aligned}$$

For the final equality to be true, there must be no contribution from the terms for which $r \neq 0$. That is, we require no aliasing at $\Omega = 0$. Since we are only interested in preserving the spectral component at $\Omega = 0$, we may sample at a rate which is lower than the Nyquist rate. The maximum value of T to satisfy these conditions is

$$T \leq \frac{1}{1 \times 10^4}.$$

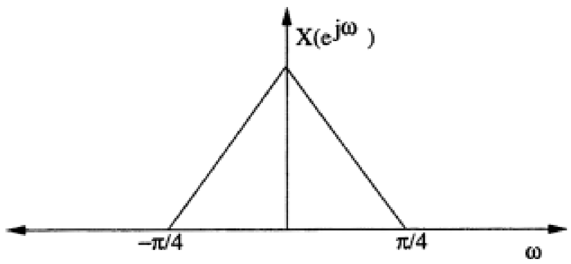
4.14. There is no loss of information if $X(e^{j\omega/2})$ and $X(e^{j(\omega/2-\pi)})$ do not overlap. This is true for (b), (d), (e).

4.15. The output $x_r[n] = x[n]$ if no aliasing occurs as result of downsampling. That is, $X(e^{j\omega}) = 0$ for $\pi/3 \leq |\omega| \leq \pi$.

(a) $x[n] = \cos(\pi n/4)$. $X(e^{j\omega})$ has impulses at $\omega = \pm\pi/4$, so there is no aliasing. $x_r[n] = x[n]$.

(b) $x[n] = \cos(\pi n/2)$. $X(e^{j\omega})$ has impulses at $\omega = \pm\pi/2$, so there is aliasing. $x_r[n] \neq x[n]$.

(c) A sketch of $X(e^{j\omega})$ is shown below. Clearly there will be no aliasing and $x_r[n] = x[n]$.



4.18. For the condition to be satisfied, we have to ensure that $\omega_0/L \leq \min(\pi/L, \pi/M)$, so that the lowpass filtering does not cut out part of the spectrum.

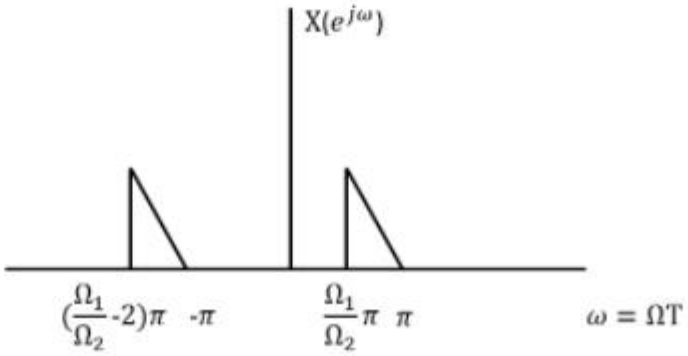
(a) $\omega_0/2 \leq \pi/3 \Longrightarrow \omega_{0,max} = 2\pi/3$.

(b) $\omega_0/3 \leq \pi/5 \Longrightarrow \omega_{0,max} = 3\pi/5$.

(c) Since $L > M$, there is no chance of aliasing. Hence $\omega_{0,max} = \pi$.

4.22

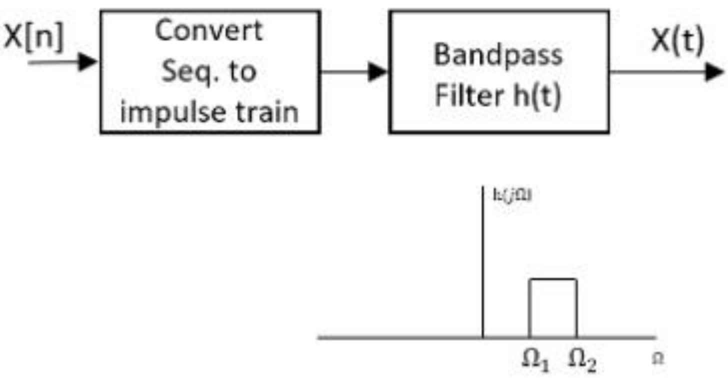
(a)



(b)

$$\text{No-aliasing} \rightarrow \Omega_s \geq 2\Omega_N$$
$$\rightarrow \Omega_s \geq \Omega_2 - \Omega_1$$

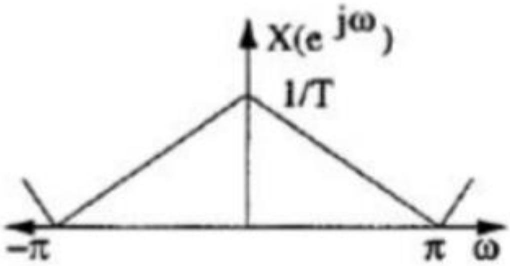
(c)



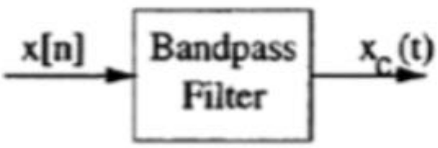
4.23

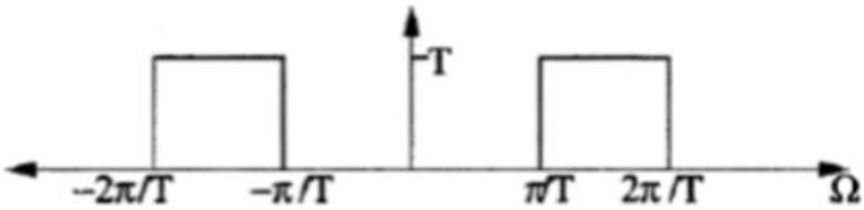
(a)

$$\omega = \Omega T, \quad T = \frac{2\pi}{\Omega_0}$$



(b) To recover simply filter out the undesired parts of $X(e^{j\omega})$.



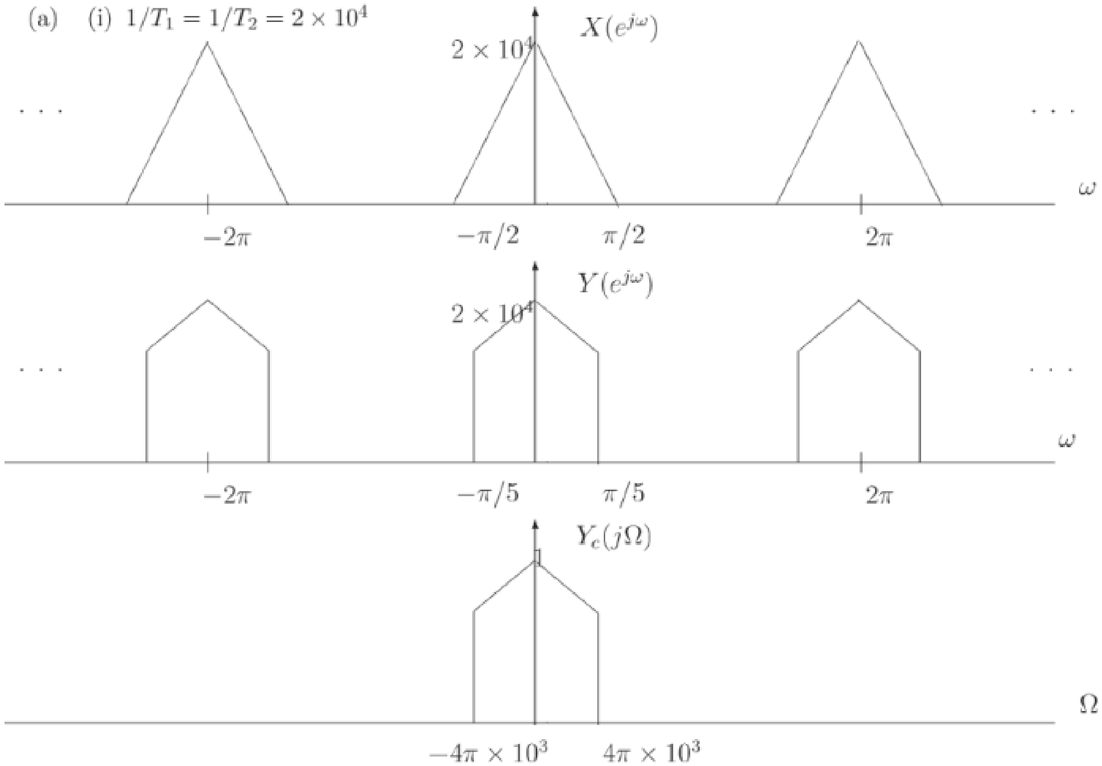


(c)

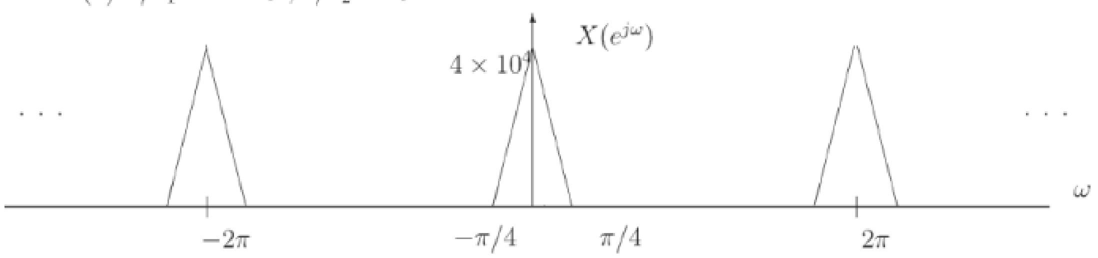
$$T \le \frac{\pi}{\Omega_0} \;\&\; T = \frac{2\pi}{\Omega_0}$$

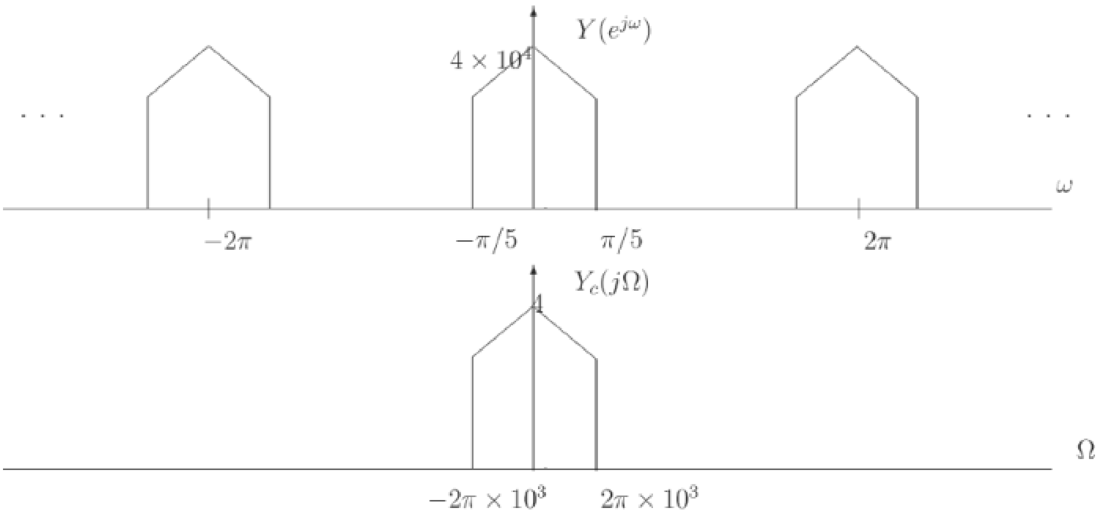
4.25

(a) (i) $1/T_1 = 1/T_2 = 2 \times 10^4$

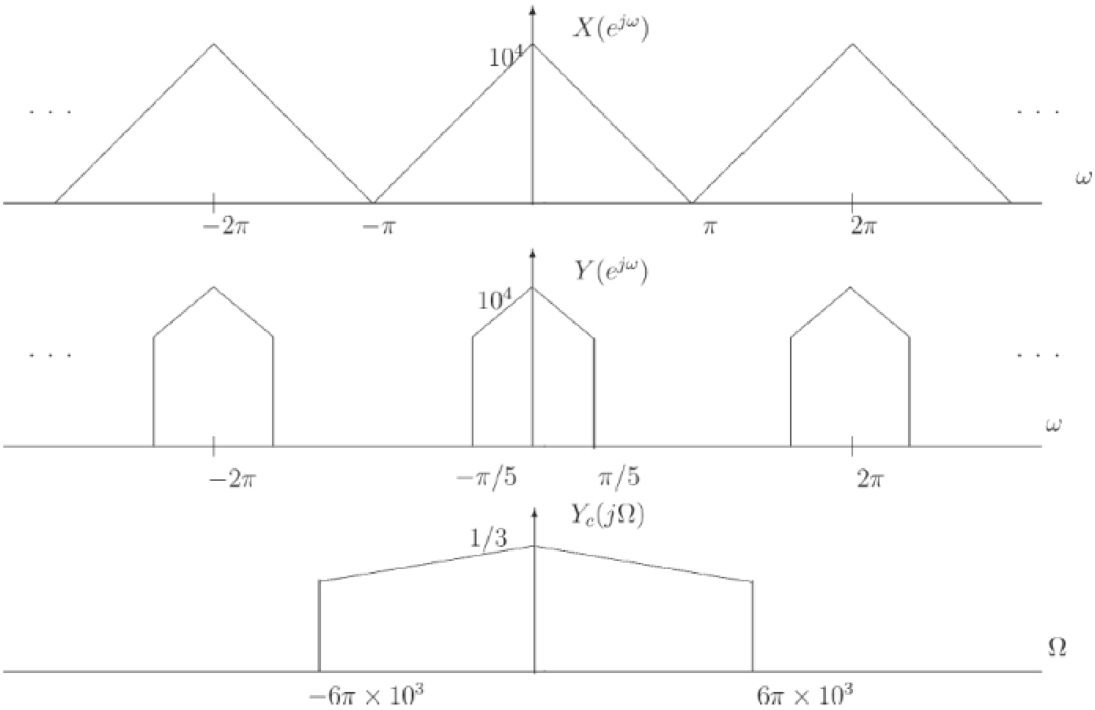


(ii) $1/T_1 = 4 \times 10^4$, $1/T_2 = 10^4$



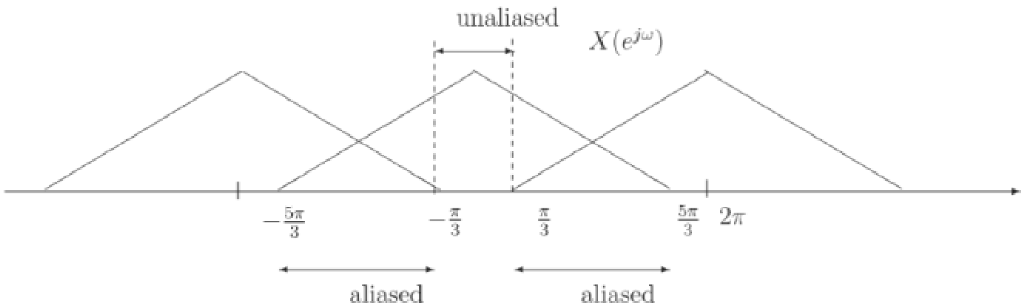


(iii) $1/T_1 = 10^4$, $1/T_2 = 3 \times 10^4$



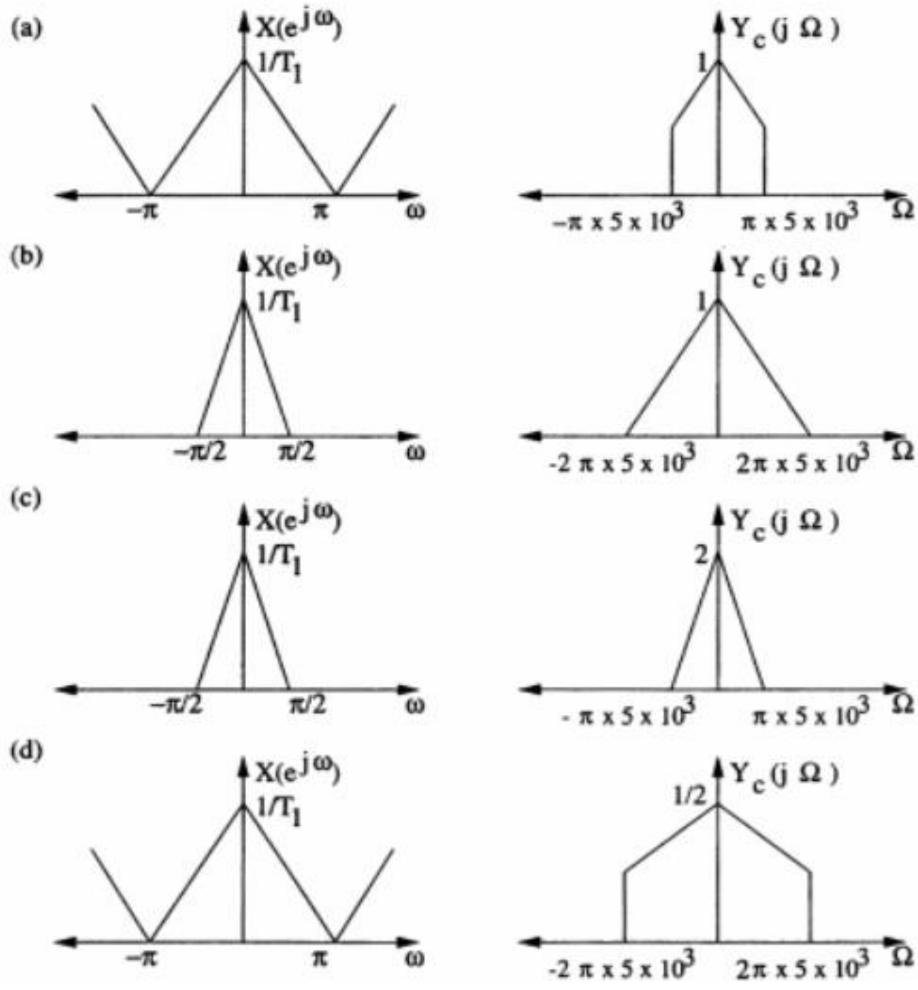
(b) From the figure below, it can be seen that the only portion of the spectrum which remains unaffected by the aliasing is $|\omega| < \pi/3$. So if we choose $\omega_c < \pi/3$, the overall system is LTI with a frequency response of

$$H_c(j\Omega) = \begin{cases} 1 & \text{for } |\Omega| < \omega_c \times 6 \times 10^3 \\ 0 & \text{otherwise.} \end{cases}$$



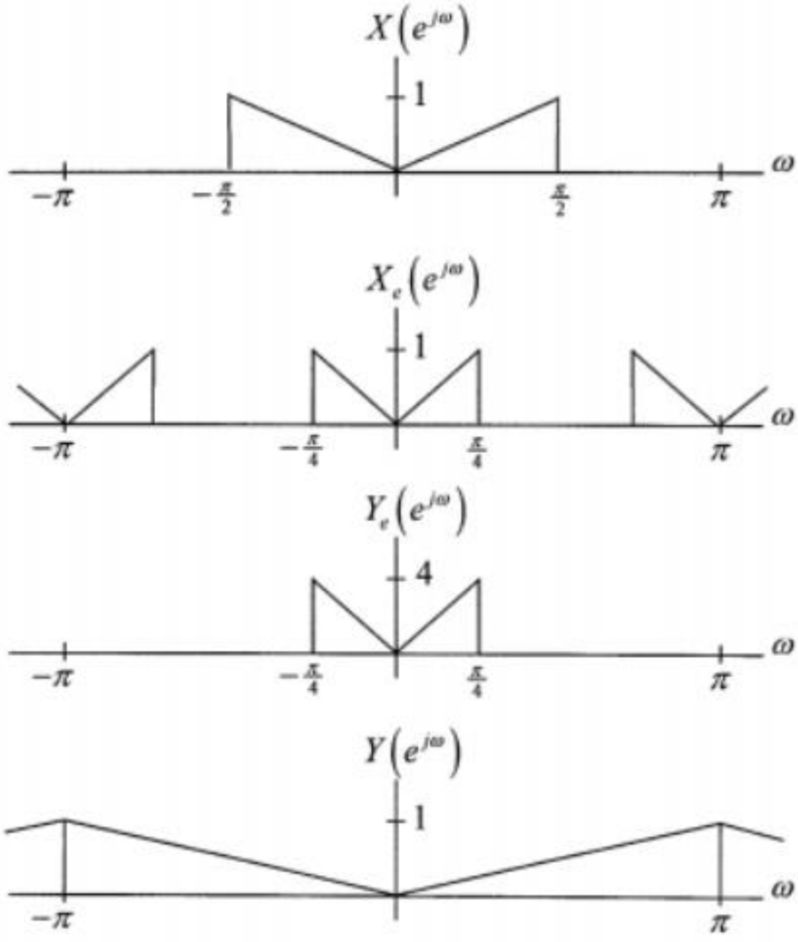
4.29

The Fourier transform of $y_c(t)$ is sketched below for each case.

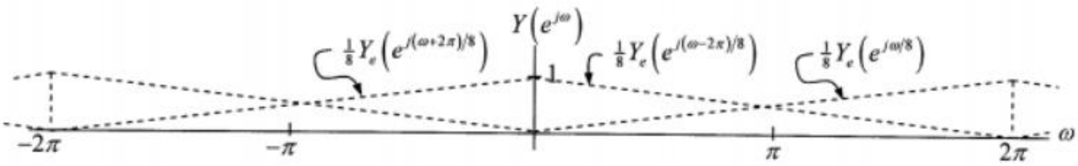


4.31

A. With $L=2$ and $M=4$,



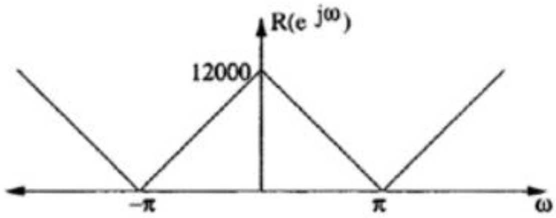
B. With $L=2$ and $M=8$, $X_e(e^{j\omega})$ and $Y_e(e^{j\omega})$ remain as in part A, except that $Y_e(e^{j\omega})$ now has a peak value of 8. After expanding we have

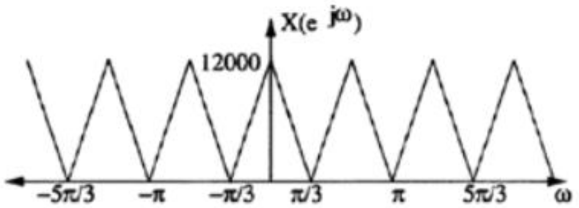


We see that $Y(e^{j\omega})=1$ for all ω. Inverse transforming gives $y[n]=\delta[n]$ in this case.

4.35

(a) See the following figure:





(b) For this to be true, $H(e^{j\omega})$ needs to filter out $X(e^{j\omega})$ for $\pi/3 \le |\omega| \le \pi$. Hence let $\omega_0 = \pi/3$. Furthermore, we want

$$\frac{\pi/2}{T_2} = 2\pi(1000) \Longrightarrow T_2 = 1/6000$$

(c) Matching the following figure of $S(e^{j\omega})$ with the figure for $R_c(j\Omega)$, and remembering that $\Omega = \omega/T$, we get $T_3 = (2\pi/3)/(2000\pi) = 1/3000$.