4.5. Consider the system of Figure 4.10, with the discrete-time system an ideal lowpass filter with cutoff frequency $\pi/8$ radians/s.

(a) If $x_c(t)$ is bandlimited to 5 kHz, what is the maximum value of T that will avoid aliasing in the C/D converter?

(b) If $1/T = 10$ kHz, what will the cutoff frequency of the effective continuous-time filter be?

(c) Repeat part (b) for $1/T = 20$ kHz.

4.7. A simple model of a multipath communication channel is indicated in Figure P4.7-1. Assume that $s_c(t)$ is bandlimited such that $S_c(j\Omega) = 0$ for $|\Omega| \geq \pi/T$ and that $x_c(t)$ is sampled with a sampling period T to obtain the sequence

$$x[n] = x_c(nT).$$

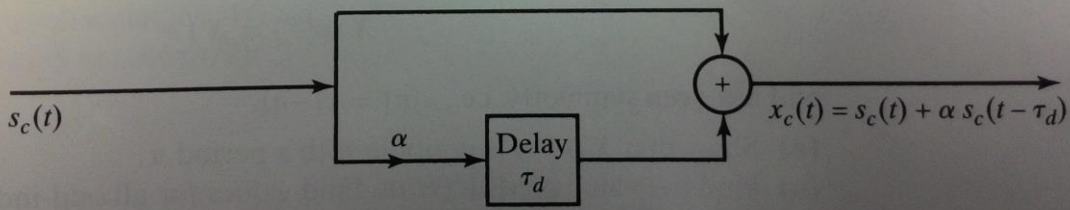


Figure P4.7-1

(a) Determine the Fourier transform of $x_c(t)$ and the Fourier transform of $x[n]$ in terms of $S_c(j\Omega)$.

(b) We want to simulate the multipath system with a discrete-time system by choosing $H(e^{j\omega})$ in Figure P4.7-2 so that the output $r[n] = x_c(nT)$ when the input is $s[n] = s_c(nT)$. Determine $H(e^{j\omega})$ in terms of T and τ_d.

(c) Determine the impulse response $h[n]$ in Figure P4.7 when (i) $\tau_d = T$ and (ii) $\tau_d = T/2$.

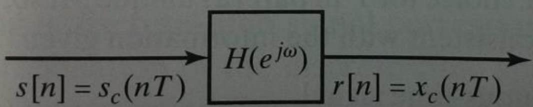


Figure P4.7-2

4.15. Consider the system shown in Figure P4.15. For each of the following input signals $x[n]$, indicate whether the output $x_r[n] = x[n]$.

(a) $x[n] = \cos(\pi n/4)$

(b) $x[n] = \cos(\pi n/2)$

(c)

$$x[n] = \left[\frac{\sin(\pi n/8)}{\pi n}\right]^2$$

Hint: Use the modulation property of the Fourier transform to find $X(e^{j\omega})$.

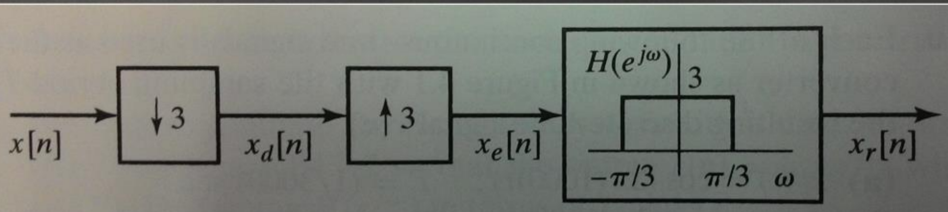


Figure P4.15

Basic Problems

4.21. Consider a continuous-time signal $x_c(t)$ with Fourier transform $X_c(j\Omega)$ shown in Figure P4.21-1.

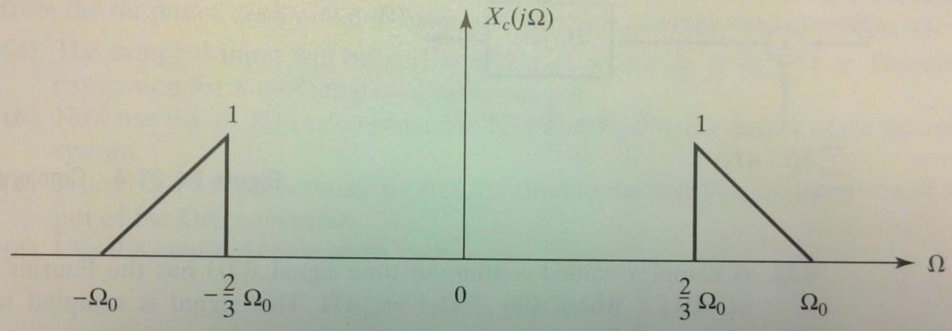


Figure P4.21-1 Fourier transform $X_c(j\Omega)$

(a) A continuous-time signal $x_r(t)$ is obtained through the process shown in Figure P4.21-2. First, $x_c(t)$ is multiplied by an impulse train of period T_1 to produce the waveform $x_s(t)$, i.e.,

$$x_s(t) = \sum_{n=-\infty}^{+\infty} x[n]\delta(t - nT_1).$$

Next, $x_s(t)$ is passed through a low pass filter with frequency response $H_r(j\Omega)$. $H_r(j\Omega)$ is shown in Figure P4.21-3.

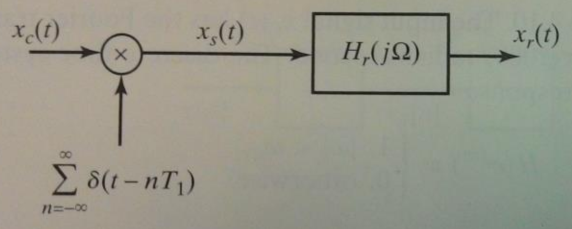


Figure P4.21-2 Conversion system for part (a)

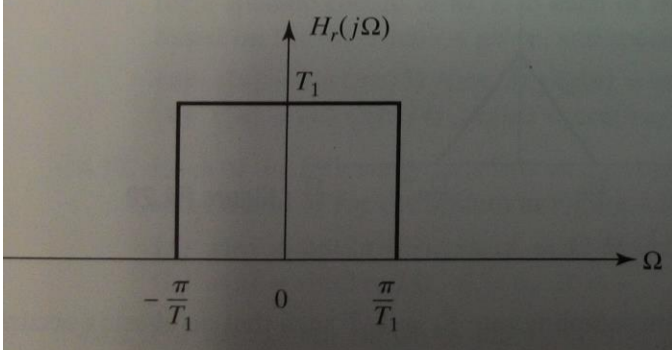


Figure P4.21-3 Frequency response $H_r(j\Omega)$

Determine the range of values for T_1 for which $x_r(t) = x_c(t)$.

(b) Consider the system in Figure P4.21-4. The system in this case is the same as the one in part (a), except that the sampling period is now T_2. The system $H_s(j\Omega)$ is some continuous-time ideal LTI filter. We want $x_o(t)$ to be equal to $x_c(t)$ for all t, i.e., $x_o(t) = x_c(t)$ for some choice of $H_s(j\Omega)$. Find all values of T_2 for which $x_o(t) = x_c(t)$ is possible. For the largest T_2 you determined that would still allow recovery of $x_c(t)$, choose $H_s(j\Omega)$ so that $x_o(t) = x_c(t)$. Sketch $H_s(j\Omega)$.

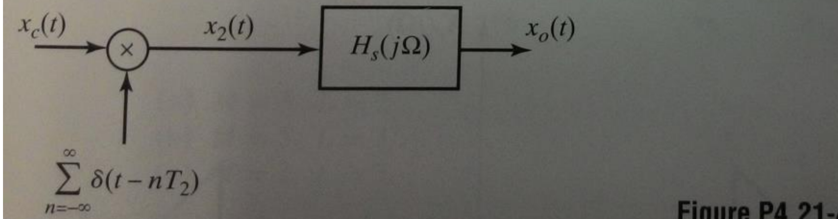


Figure P4.21-4 Conversion system for part (b)

4.23. A continuous-time signal $x_c(t)$, with Fourier transform $X_c(j\Omega)$ shown in Figure P4.23, is sampled with sampling period $T = 2\pi/\Omega_0$ to form the sequence $x[n] = x_c(nT)$.

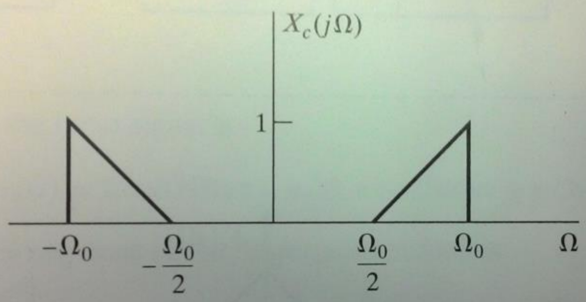


Figure P4.23

(a) Sketch the Fourier transform $X(e^{j\omega})$ for $|\omega| < \pi$.

(b) The signal $x[n]$ is to be transmitted across a digital channel. At the receiver, the original signal $x_c(t)$ must be recovered. Draw a block diagram of the recovery system and specify its characteristics. Assume that ideal filters are available.

(c) In terms of Ω_0, for what range of values of T can $x_c(t)$ be recovered from $x[n]$?

4.25. Figure P4.25-1 shows a continuous-time filter that is implemented using an LTI discrete-time filter ideal lowpass filter with frequency response over $-\pi \leq \omega \leq \pi$ as

$$H(e^{j\omega}) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| \leq \pi. \end{cases}$$

(a) If the continuous-time Fourier transform of $x_c(t)$, namely $X_c(j\Omega)$, is as shown in Figure P4.25-2 and $\omega_c = \frac{\pi}{5}$, sketch and label $X(e^{j\omega})$, $Y(e^{j\omega})$ and $Y_c(j\Omega)$ for each of the following cases:

(i) $1/T_1 = 1/T_2 = 2 \times 10^4$

(ii) $1/T_1 = 4 \times 10^4,\ \ 1/T_2 = 10^4$

(iii) $1/T_1 = 10^4,\ \ 1/T_2 = 3 \times 10^4$.

(b) For $1/T_1 = 1/T_2 = 6 \times 10^3$, and for input signals $x_c(t)$ whose spectra are bandlimited to $|\Omega| < 2\pi \times 5 \times 10^3$ (but otherwise unconstrained), what is the maximum choice of the cutoff frequency ω_c of the filter $H(e^{j\omega})$ for which the overall system is LTI? For this maximum choice of ω_c, specify $H_c(j\Omega)$.

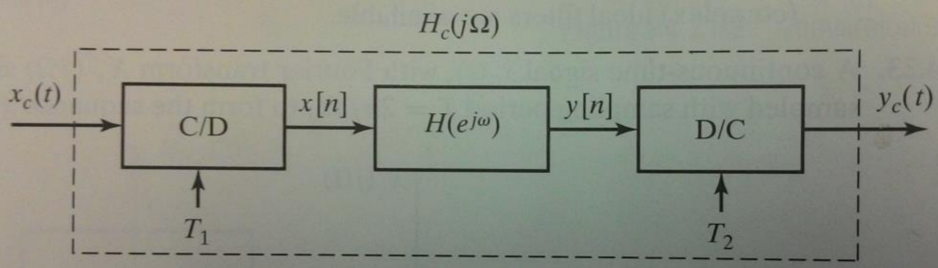


Figure P4.25-1

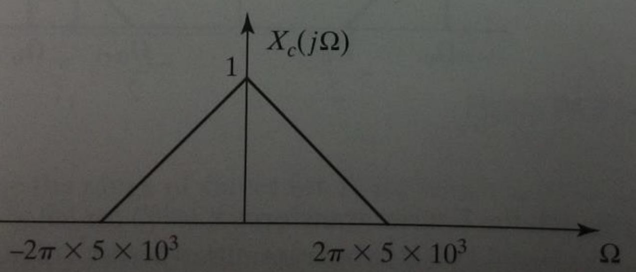


Figure P4.25-2

4.27. Consider the representation of the process of sampling followed by reconstruction shown in Figure P4.27.

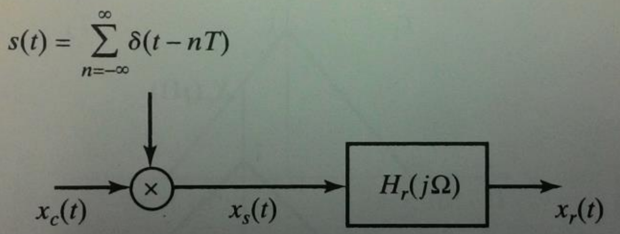


Figure P4.27

Assume that the input signal is

$$x_c(t) = 2\cos(100\pi t - \pi/4) + \cos(300\pi t + \pi/3) \qquad -\infty < t < \infty$$

The frequency response of the reconstruction filter is

$$H_r(j\Omega) = \begin{cases} T & |\Omega| \leq \pi/T \\ 0 & |\Omega| > \pi/T \end{cases}$$

(a) Determine the continuous-time Fourier transform $X_c(j\Omega)$ and plot it as a function of Ω.

(b) Assume that $f_s = 1/T = 500$ samples/sec and plot the Fourier transform $X_s(j\Omega)$ as a function of Ω for $-2\pi/T \leq \Omega \leq 2\pi/T$. What is the output $x_r(t)$ in this case? (You should be able to give an exact equation for $x_r(t)$.)

(c) Now, assume that $f_s = 1/T = 250$ samples/sec. Repeat part (b) for this condition.

(d) Is it possible to choose the sampling rate so that

$$x_r(t) = A + 2\cos(100\pi t - \pi/4)$$

where A is a constant? If so, what is the sampling rate $f_s = 1/T$, and what is the numerical value of A?

4.29. In the system of Figure P4.29, $X_c(j\Omega)$ and $H(e^{j\omega})$ are as shown. Sketch and label the Fourier transform of $y_c(t)$ for each of the following cases:

(a) $1/T_1 = 1/T_2 = 10^4$
(b) $1/T_1 = 1/T_2 = 2 \times 10^4$
(c) $1/T_1 = 2 \times 10^4, \quad 1/T_2 = 10^4$
(d) $1/T_1 = 10^4, \quad 1/T_2 = 2 \times 10^4$.

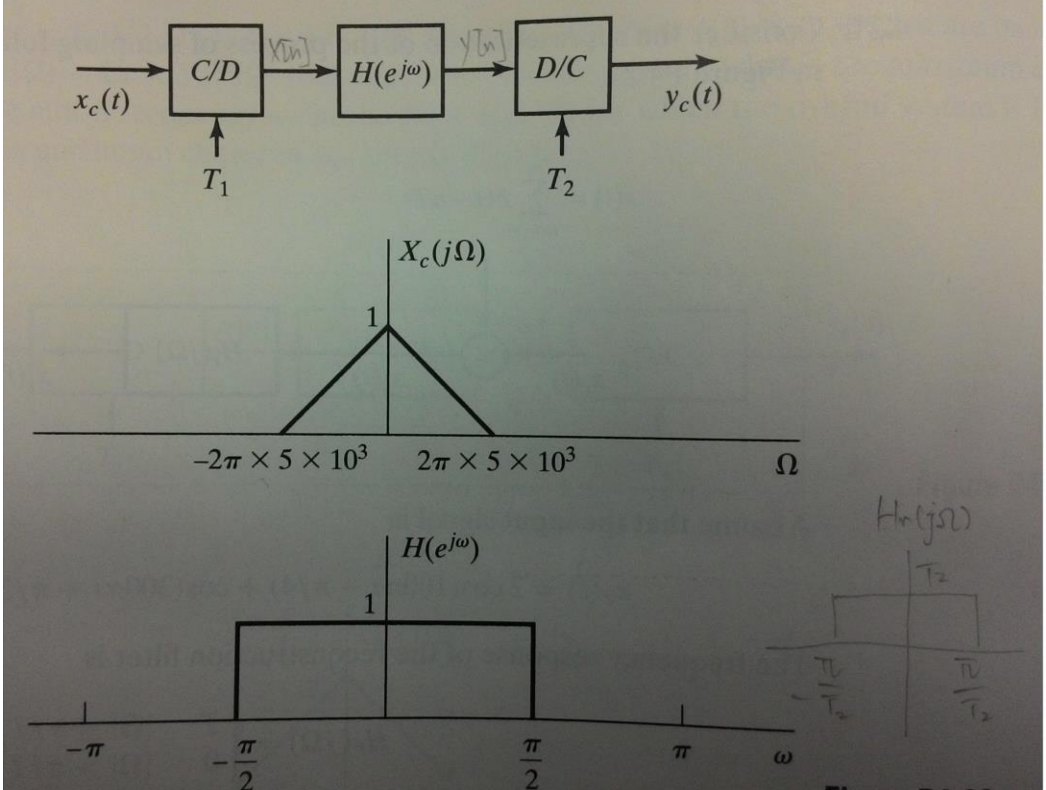


Figure P4.29

4.31. Consider the discrete-time system shown in Figure P4.31-1

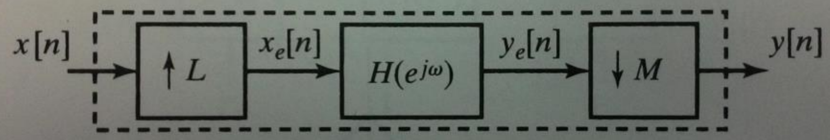


Figure P4.31-1

where

(i) L and M are positive integers.

(ii) $x_e[n] = \begin{cases} x[n/L] & n = kL, \quad k \text{ is any integer} \\ 0 & \text{otherwise.} \end{cases}$

(iii) $y[n] = y_e[nM]$.

(iv) $H(e^{j\omega}) = \begin{cases} M & |\omega| \le \frac{\pi}{4} \\ 0 & \frac{\pi}{4} < |\omega| \le \pi . \end{cases}$

(a) Assume that $L = 2$ and $M = 4$, and that $X(e^{j\omega})$, the DTFT of $x[n]$, is real and is shown in Figure P4.31-2. Make an appropriately labeled sketch of $X_e(e^{j\omega})$, $Y_e(e^{j\omega})$, and $Y(e^{j\omega})$, the DTFTs of $x_e[n]$, $y_e[n]$, and $y[n]$, respectively. Be sure to clearly label salient amplitudes and frequencies.

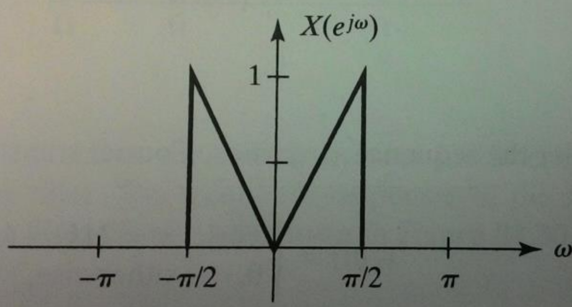
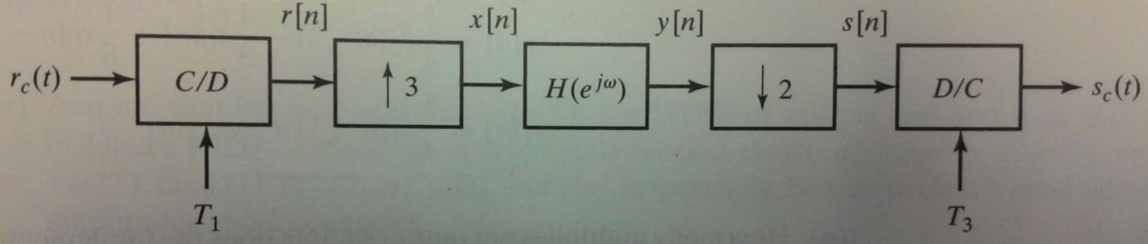


Figure P4.31-2

(b) Now assume $L = 2$ and $M = 8$. Determine $y[n]$ in this case.
Hint: See which diagrams in your answer to part (a) change.

4.35. Consider the system given in Figure P4.35. You may assume that $R_c(j\Omega)$ is bandlimited; i.e., $R_c(j\Omega) = 0, \quad |\Omega| \geq 2\pi(1000)$, as shown in the figure.



$$T_1 = \frac{1}{2000} \text{ seconds} \qquad H(e^{j\omega}) = \begin{cases} 1, 0 \leq \omega \leq \omega_0 \\ 0, \omega_0 < \omega \leq \pi \end{cases}$$

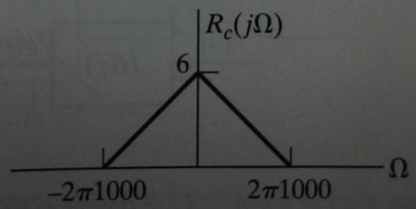


Figure P4.35

(a) Sketch $R(e^{j\omega})$ and $X(e^{j\omega})$.

(b) Choose nonzero values for ω_0 and T_2 such that

$$y[n] = \alpha r_c(nT_2)$$

for some nonzero constant α. (You do not have to determine the value of α.)

(c) Using the value of ω_0 you obtained in Part (b), determine a choice for T_3 such that

$$s_c(t) = \beta r_c(t)$$

for some nonzero constant β. (You do not have to determine the value of β.)

4.37. Consider the decimation filter structure shown in Figure P4.37-1:

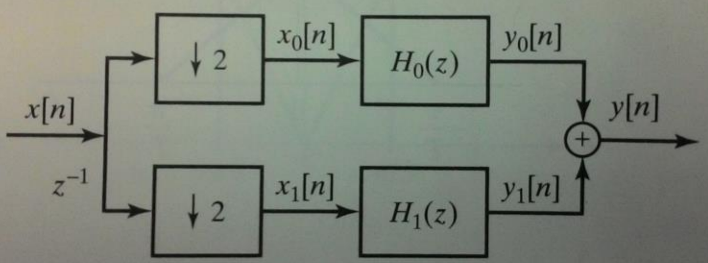


Figure P4.37-1

where $y_0[n]$ and $y_1[n]$ are generated according to the following difference equations:

$$y_0[n] = \frac{1}{4}y_0[n-1] - \frac{1}{3}x_0[n] + \frac{1}{8}x_0[n-1]$$

$$y_1[n] = \frac{1}{4}y_1[n-1] + \frac{1}{12}x_1[n]$$

where $y_0[n]$ and $y_1[n]$ are generated according to the following difference equations:

$$y_0[n] = \frac{1}{4}y_0[n-1] - \frac{1}{3}x_0[n] + \frac{1}{8}x_0[n-1]$$

$$y_1[n] = \frac{1}{4}y_1[n-1] + \frac{1}{12}x_1[n]$$

(a) How many multiplies per output sample does the implementation of the filter structure require? Consider a divide to be equivalent to a multiply.

The decimation filter can also be implemented as shown in Figure P4.37-2,

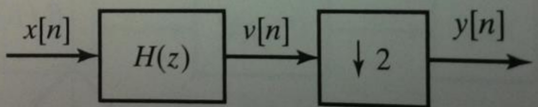


Figure P4.37-2

where $v[n] = av[n-1] + bx[n] + cx[n-1]$.

(b) Determine a, b, and c.

(c) How many multiplies per output sample does this second implementation require?

4.46. Consider the system shown in Figure P4.46-1 for discrete-time processing of the continuous-time input signal $g_c(t)$. The input signal $g_c(t)$ is of the form $g_c(t) = f_c(t) + e_c(t)$, where the Fourier transforms of $f_c(t)$ and $e_c(t)$ are shown in Figure P4.46-2. Since the input signal is not bandlimited, a continuous-time antialiasing filter $H_{aa}(j\Omega)$ is used. The magnitude of the frequency response for $H_{aa}(j\Omega)$ is shown in Figure P4.46-3, and the phase response of the antialiasing filter is $\angle H_{aa}(j\Omega) = -\Omega^3$.

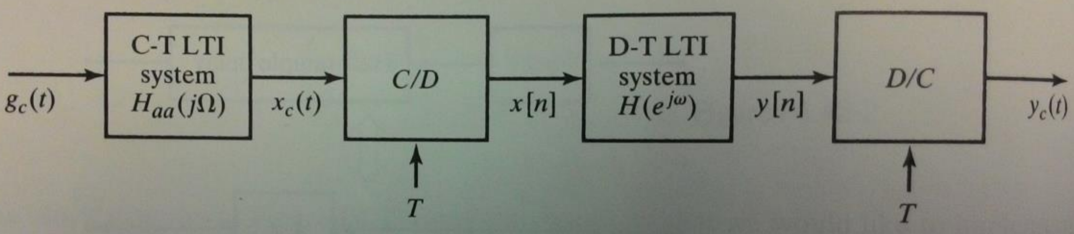


Figure P4.46-1

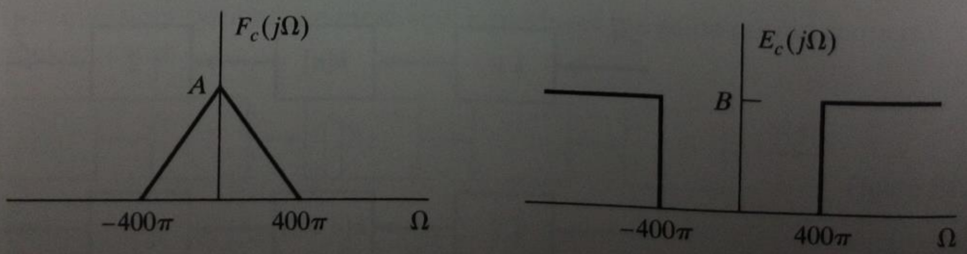


Figure P4.46-2

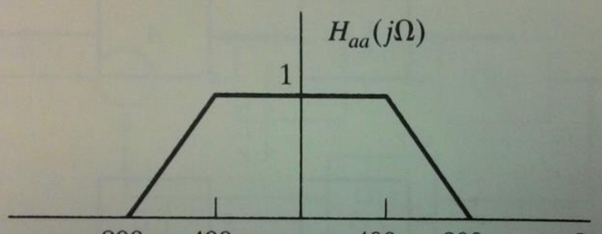


Figure P4.46-3

(a) If the sampling rate is $2\pi/T = 1600\pi$, determine the magnitude and phase of $H(e^{j\omega})$, the frequency response of the discrete-time system, so that the output is $y_c(t) = f_c(t)$.

(b) Is it possible that $y_c(t) = f_c(t)$ if $2\pi/T < 1600\pi$? If so, what is the *minimum* value of $2\pi/T$? Determine $H(e^{j\omega})$ for this choice of $2\pi/T$.