Principles of Communications Lecture 8: Baseband Communication Systems

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Outlines

- Introduction
- Line codes
- Effects of filtering
- Pulse shaping toward zero ISI
- Zero-forcing equalization
- Eye diagrams
- Synchronization

Introduction

- Digital vs. analog signals
- Analog → sampling → quantization → digital
- Baseband vs. passband
- Channel distortion – ISI

  (Channel) bandwidth limitation

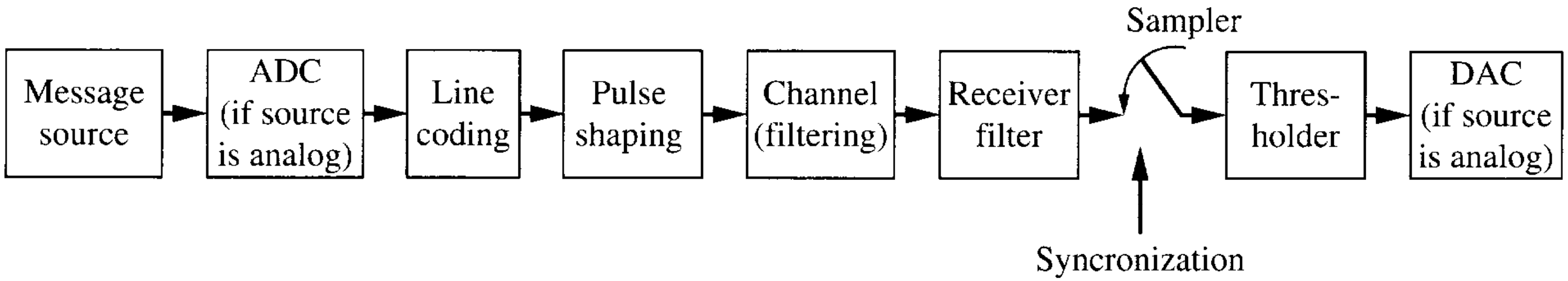


Figure 4.1
Block diagram of a baseband digital data transmission system.

Line Codes

- Baseband *data format* used to represent digital data (for transmission purpose).
- Examples are given on the next page
- Operation: time or frequency *shaping*
- Purposes: usually to cope with the channel limitations (or provide extra function such as synchronization)
- Needed for certain applications

- Non-return-to-zero (NRZ) change
- NRZ mark (data1-> change in level; data0 -> no change)
- Unipolar return-to-zero (URZ) <1/2 width pulses>
- Polar RZ
- Bipolar RZ ("0"-> 0 level; "1"-> alternate sign
- Split phase (Manchester) ("1" -> level A to level –A at 1/2 interval; "0" -> level –A to level A at 1/2 interval)

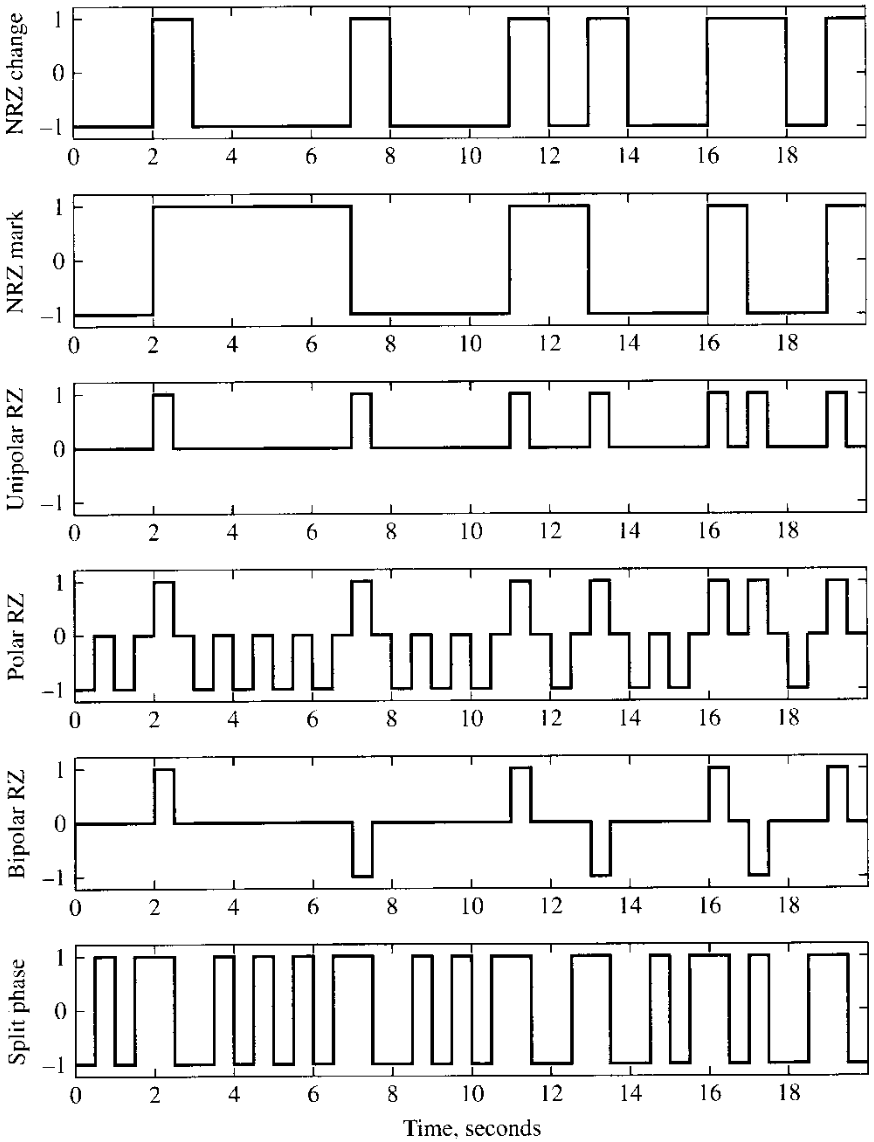


Figure 4.2
Abbreviated list of binary data formats.

Purposes of Line Codes

- **Self synchronization**
- **Proper power spectrum**
- **Transmission bandwidth**
- Transparency
- Error detection capability
- **Good error probability** performance

Power Spectra (I)

The transmitted signal is a pulse train:

$$X(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT - \Delta).$$

The amplitudes can be viewed as random variables with

$$R_m = \langle a_k a_{k+m} \rangle \qquad m = 0,\ \pm 1,\ \pm 2,\ \ldots$$

The autocorrelation function of the waveform is

$$R_X(\tau) = \sum_{m=-\infty}^{\infty} R_m r(\tau - mT),$$

in which $r(\tau) = \dfrac{1}{T}\int_{-\infty}^{\infty} p(t+\tau)p(t)dt.$

Power Spectra (II)

The power spectral density is the Fourier transofrm of $R_X(\tau)$:

$$S_X(f) = \mathbf{FT}[R_X(\tau)] = \mathbf{FT}[\sum_{m=-\infty}^{\infty} R_m r(\tau - mT)]$$

$$= \sum_{m=-\infty}^{\infty} R_m \mathbf{FT}[r(\tau - mT)] = \sum_{m=-\infty}^{\infty} R_m S_r(f) e^{-j2\pi mTf}$$

$$= S_r(f) \sum_{m=-\infty}^{\infty} R_m e^{-j2\pi mTf}.$$

Note that $S_r(f) = \mathbf{FT}[r(\tau)] = \mathbf{FT}[\frac{1}{T} p(-t) * p(t)] = \frac{|P(f)|^2}{T}.$

Example 1: NRZ

- Assume the message $m[n]$ is random (white noise) with equally "0" and "1" values.
- Step 1: Compute R_m based on the above assumption and "format"
- Step 2: Compute $r(t)$ based on the pulse shape.

1. NRZ, 50% "0" and 50% "1".

$$R_m = \tfrac{1}{2}A^2 + \tfrac{1}{2}(-A)^2 = A^2, m = 0;$$

$$R_m = \tfrac{1}{4}A^2 + \tfrac{1}{4}A(-A) + \tfrac{1}{4}(-A)A + \tfrac{1}{2}(-A)^2 = 0 \ , m \neq 0.$$

$$p(t) = \Pi(t/T) \rightarrow P(f) = T\text{sinc}(Tf)$$

Therefore, $S_{NRZ}(f) = A^2 S_r(f) = A^2 T\text{sinc}^2(Tf).$

Example 2: Unipolar RZ

2. Unipolar RZ, 50% 1-level and 50% 0-level.

$$R_m = \begin{cases} \frac{1}{2}A^2 + \frac{1}{2}(0)^2 = \frac{1}{2}A^2, & m = 0 \\ \frac{1}{4}A \cdot A + \frac{1}{4}A \cdot 0 + \frac{1}{4}0 \cdot A + \frac{1}{4}0 \cdot 0 = \frac{1}{4}A^2, & m \neq 0 \end{cases}$$

$$p(t) = \Pi(2t/T) \rightarrow P(f) = \tfrac{T}{2}\text{sinc}(\tfrac{T}{2}f)$$

$$S_{URZ}(f) = \frac{T}{4}\text{sinc}^2(\frac{T}{2}f)\left[\frac{1}{2}A^2 + \frac{1}{4}A^2 \sum_{m=-\infty, m\neq 0}^{m=\infty} e^{-j2\pi mTf}\right]$$

$$= \frac{T}{4}\text{sinc}^2(\frac{T}{2}f)\left[\frac{1}{4}A^2 + \frac{1}{4}A^2 \sum_{m=-\infty}^{m=\infty} e^{-j2\pi mTf}\right]$$

$$= \frac{T}{4}\text{sinc}^2(\frac{T}{2}f)\left[\frac{1}{4}A^2 + \frac{1}{4}\frac{A^2}{T} \sum_{n=-\infty}^{n=\infty} \delta(f - \frac{n}{T})\right]. \because \sum_{m=-\infty}^{m=\infty} e^{-j2\pi mTf} = \frac{1}{T}\sum_{n=-\infty}^{n=\infty}\delta(f - \frac{n}{T})$$

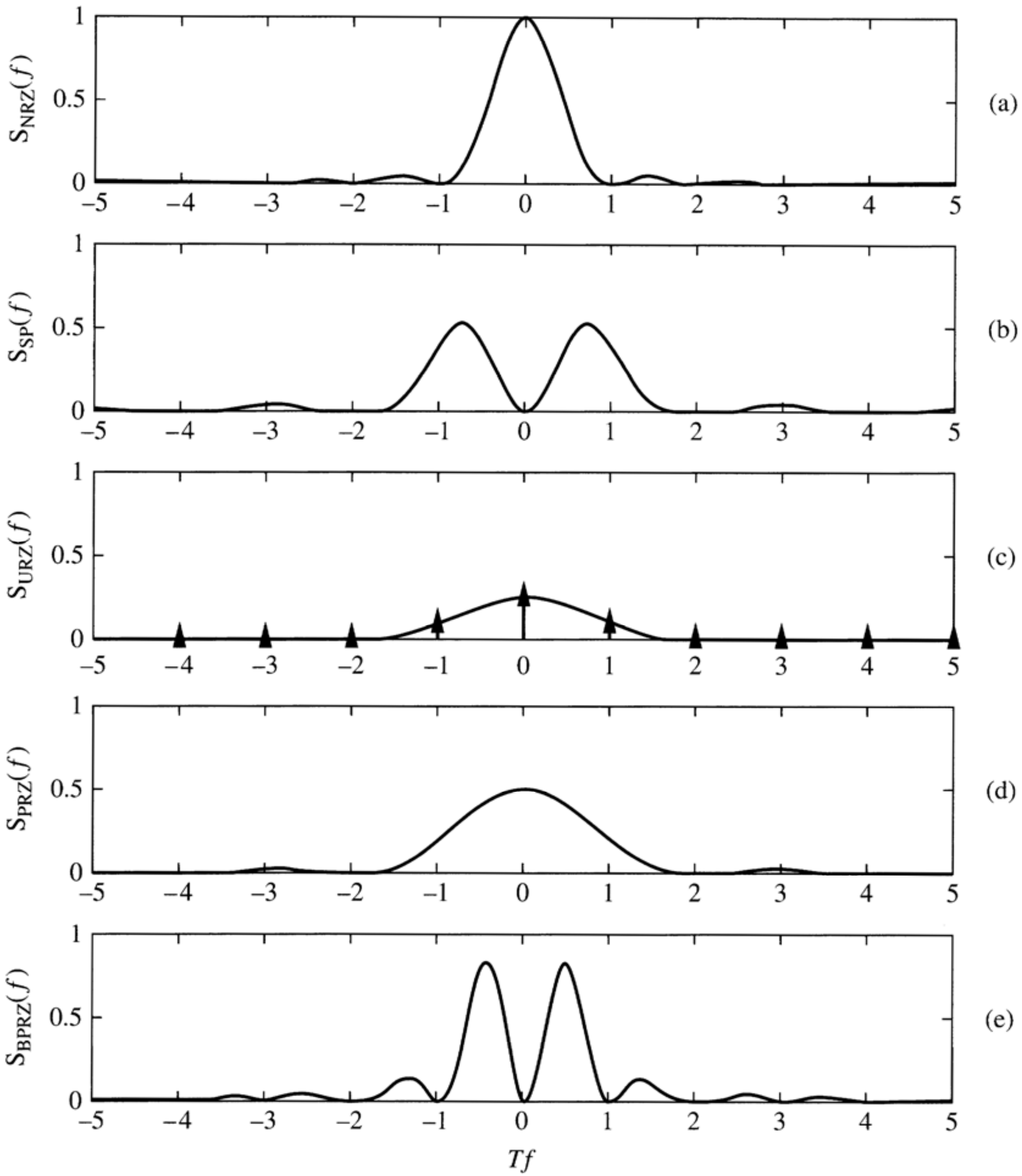


Figure 4.3
Power spectra for line-coded binary data formats.

Inter-symbol Interference (ISI)

- **Intersymbol interference** refers to one specific type of *distortion*. It is typically due to insufficient bandwidth of the channel. *Note*: Narrow BW → Long time-duration

- Example: Rectangular waveforms through a lowpass RC filter. The neighboring pulses smear out and interfere with each other.

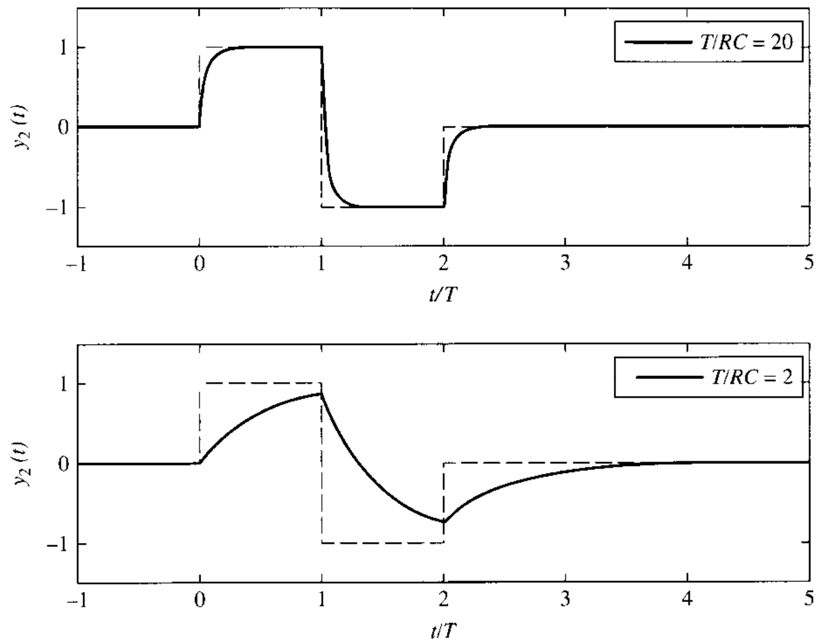
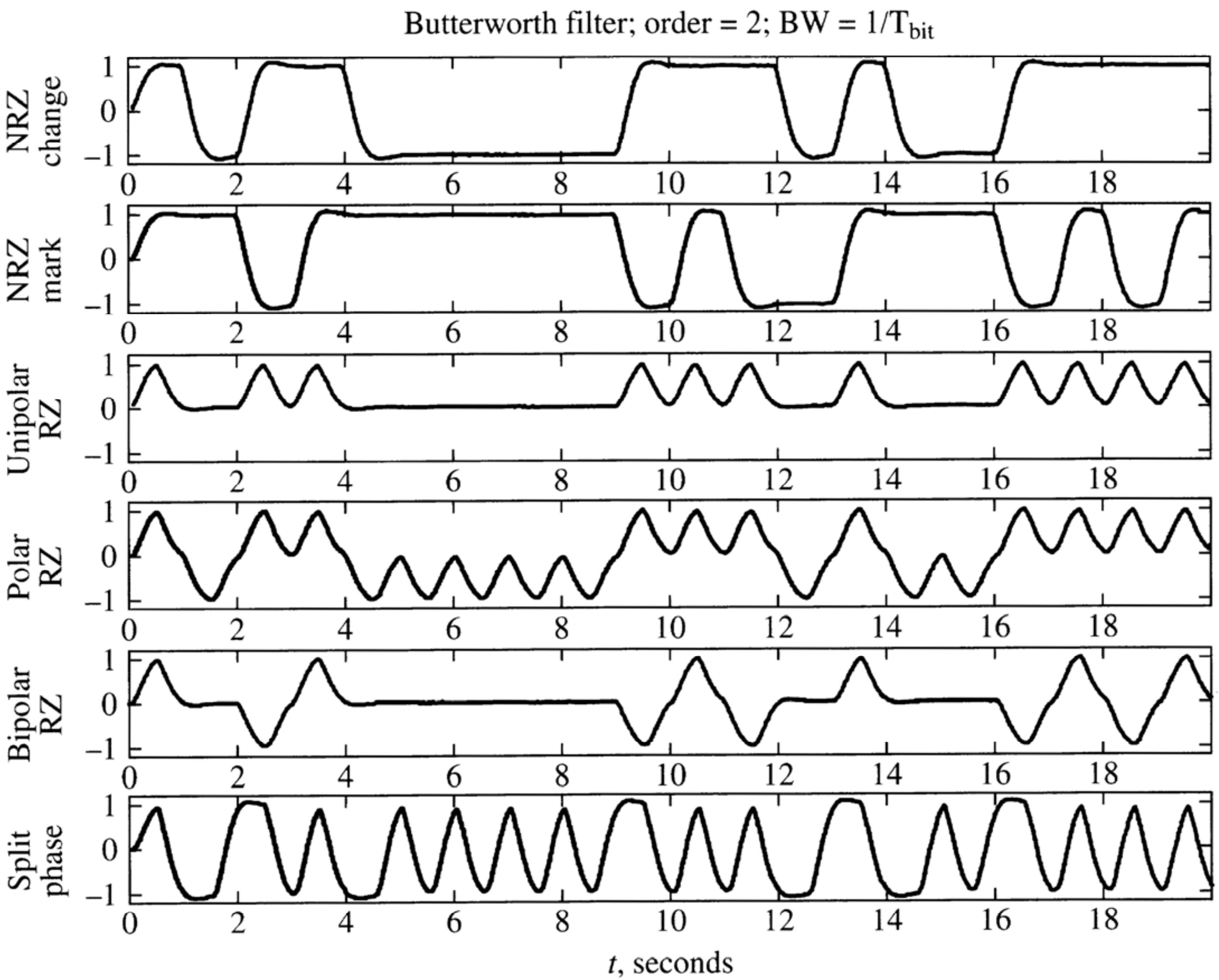
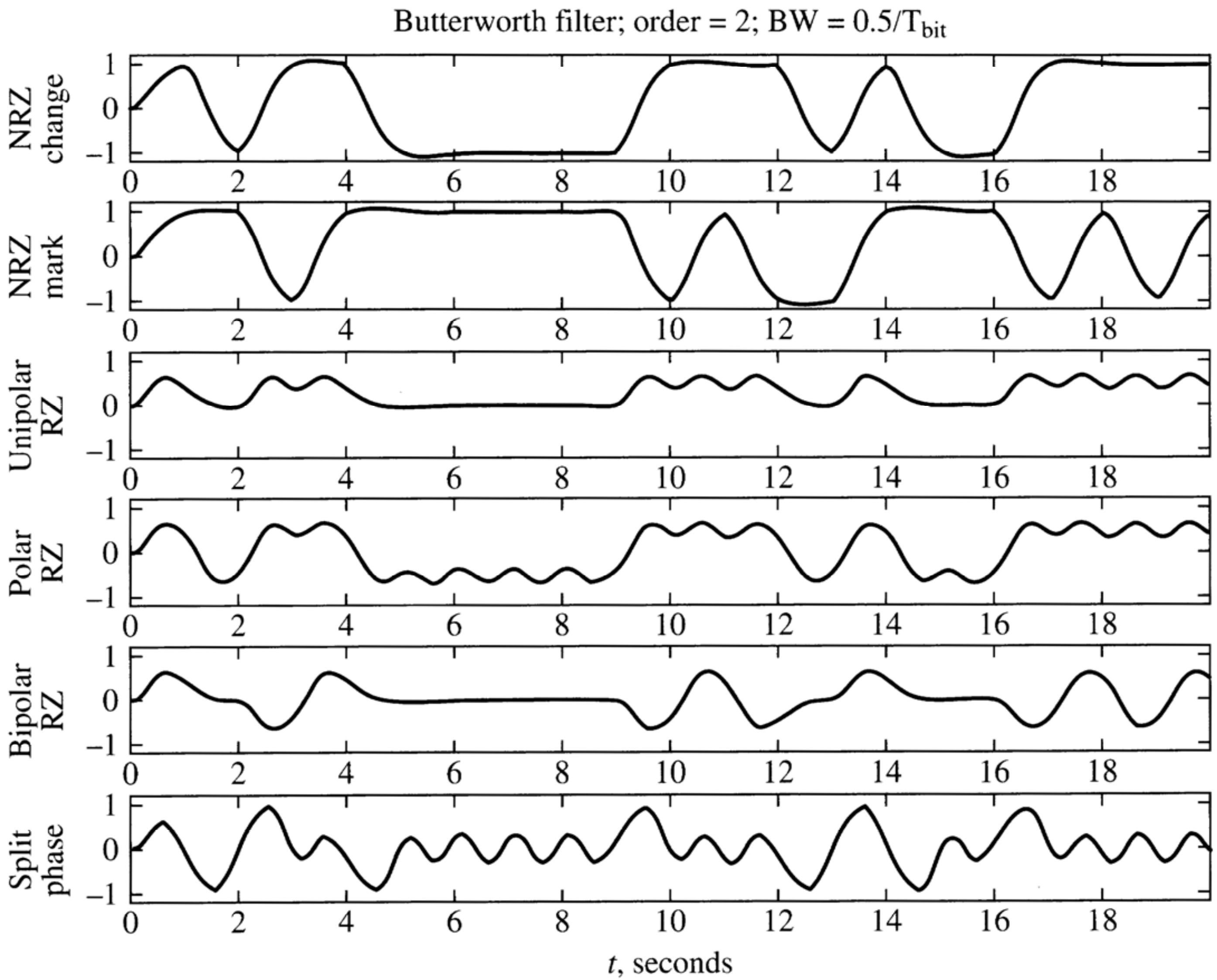


Figure 4.4
Response of a lowpass RC filter to a positive rectangular pulse followed by a negative rectangular pulse to illustrate the concept of ISI. (a) $T/RC = 20$. (b) $T/RC = 2$.



BW=
1bit/sec

Figure 4.5
Data sequences formatted with various line codes passed through a channel represented by a second-order lowpass Butterworth filter of bandwidth 1 bit rate.



BW= 0.5bit/sec

Figure 4.6
Data sequences formatted with various line codes passed through a channel represented by a second-order lowpass Butterworth filter of bandwidth $\frac{1}{2}$ bit rate.

ISI Reduction

- **Pulse shaping**: Assume the channel is ideal (flat) with the minimum BW (W). What shape of “pulse” warrants ISI-free transmission?
- **Equalization**: If the channel is non-ideal (not ideally flat LPF), an equalizer helps in reducing ISI.

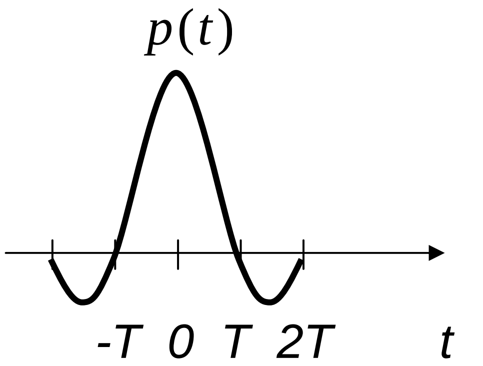
Pulse Shaping

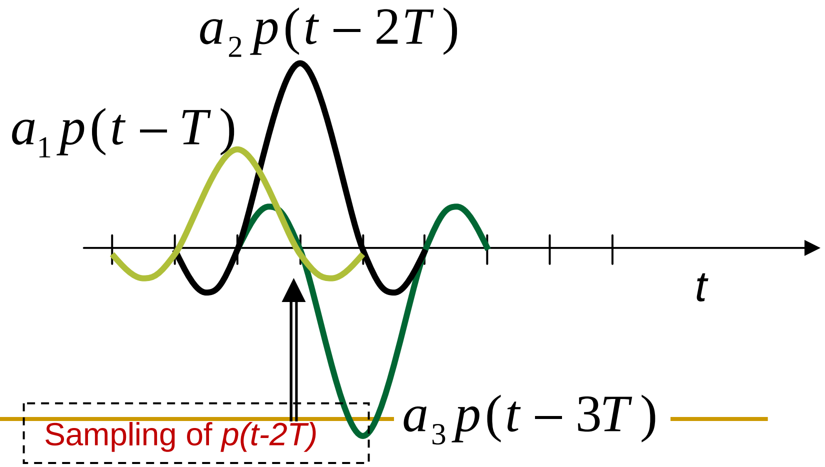
- Consider 2*W* independent samples per second are transmitted through a channel with bandwidth *W* Hz. The output is as follows.

$$y(t)=\sum_{n=-\infty}^{\infty} y_n(t)=\sum_{n=-\infty}^{\infty} a_n \operatorname{sinc}\left[2W\left(t-\frac{n}{2W}\right)\right].$$

The samples at $t_m = m/2W$ are ISI-free, $\because$ $\operatorname{sinc}(m-n)=0.$

- Are there other pulse shapes warrant ISI-free?





Raised Cosine Family

- **Raised Cosine**: a example of ISI-free pulses – raised cosine spectra at BW edges.

$$P_{RC}(f)=\begin{cases} T, & |f|\le \dfrac{1-\beta}{2T} \\ \dfrac{T}{2}\left\{1+\cos\left[\dfrac{\pi T}{\beta}\left(|f|-\dfrac{1-\beta}{2T}\right)\right]\right\} & \dfrac{1-\beta}{2T}<|f|\le \dfrac{1+\beta}{2T} \\ 0 & |f|>\dfrac{1+\beta}{2T} \end{cases}$$

- **Roll-off factor** β : β=0 → rectangular pulse (sinc)

  β=1 → “cosine” in freq; time pulse has narrow main lobe with very low sidelobes

Time pulse of the raised cosine:

$$p_{RC}(t) = \frac{\cos(\pi\beta t / T)}{1-(2\beta t / T)^2}\,\mathrm{sinc}(\frac{t}{T}), \beta \text{ is called the roll-off factor.}$$

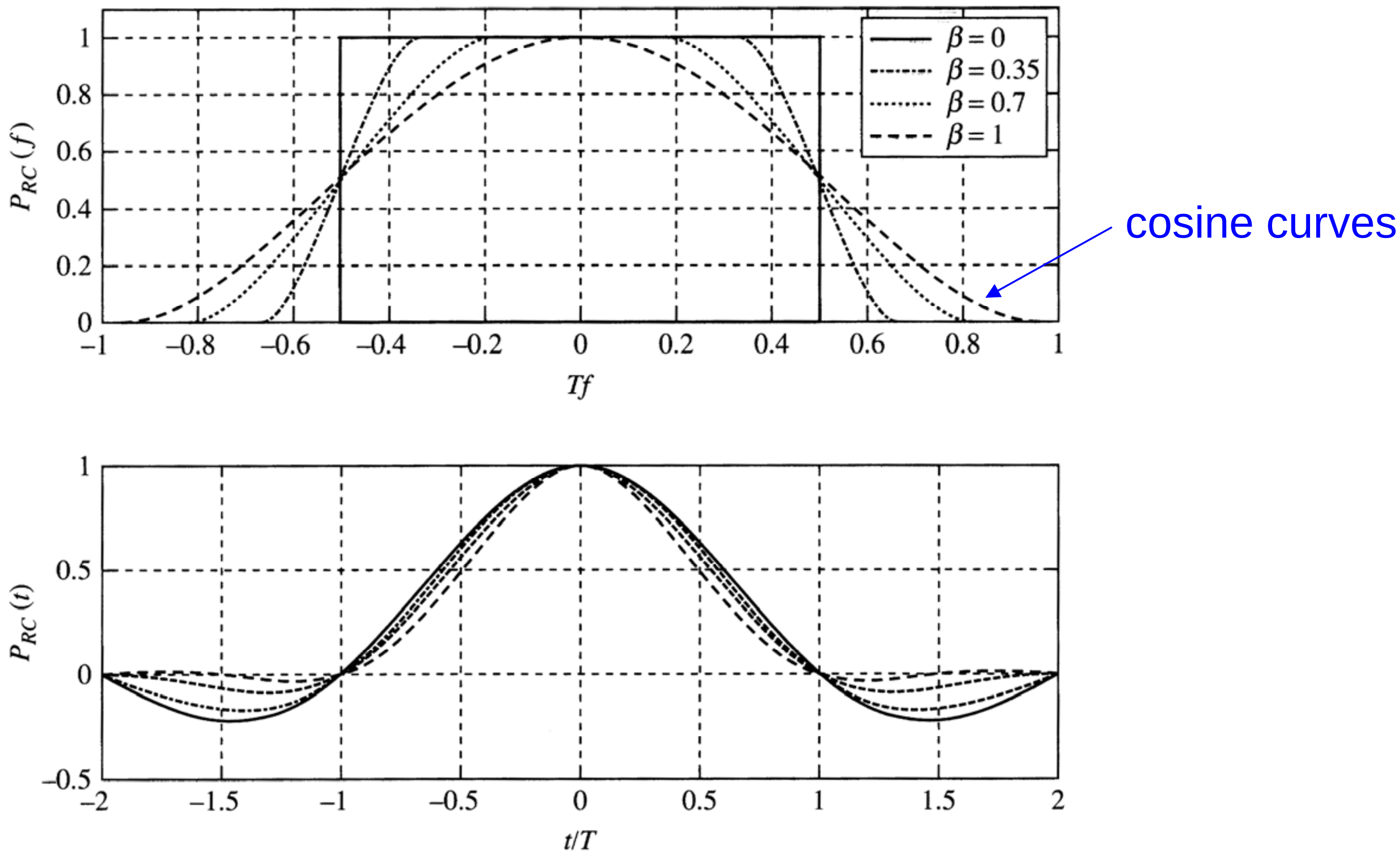


Figure 4.7
(a) Raised cosine spectra and (b) corresponding pulse responses.

Nyquist's Pulse Shaping Criterion

A pulse shape $p(t)$ with a Fourier transform $P(f)$ satisfying

$$\sum_{k=-\infty}^{\infty} P(f+\frac{k}{T}) = T, \qquad |f| \le \frac{1}{2T}$$

then its sampled values $p(nT) = \begin{cases} 1, & n=0 \\ 0, & n \ne 0 \end{cases}.$

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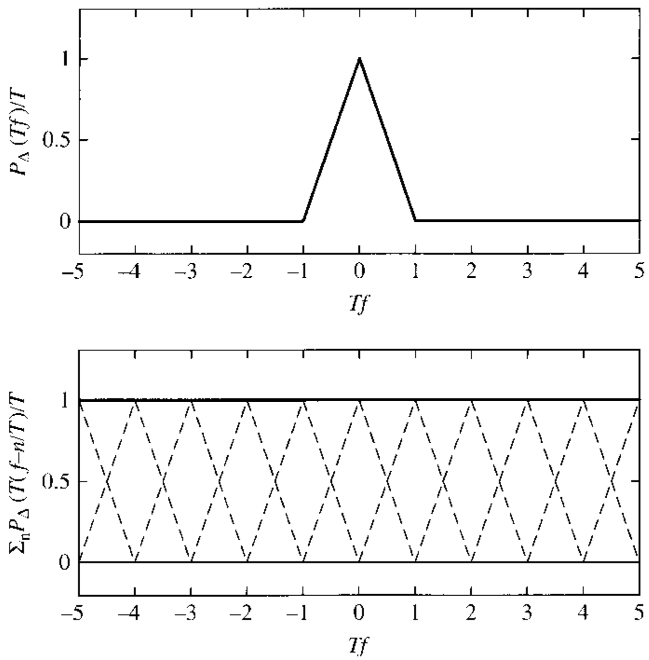


Figure 4.8
Illustration that (a) a triangular spectrum satisfies (b) Nyquist's zero ISI criterion.

Proof of Nyquist's Pulse Shaping Criterion

$$p(t) = \int_{-\infty}^{\infty} P(f) e^{j2\pi ft} df$$

Sampling at nT, $p(nT) = \int_{-\infty}^{\infty} P(f) e^{j2\pi fnT} df$

$$p(nT) = \sum_{k=-\infty}^{\infty} \int_{2k-1/2T}^{2k+1/2T} P(f) e^{j2\pi fnT} df$$

$$= \sum_{k=-\infty}^{\infty} \int_{-1/2T}^{1/2T} P(u + \frac{k}{T}) e^{j2\pi f(unT+kn)} du; \quad u = f - \frac{k}{T}$$

$$= \int_{-1/2T}^{1/2T} \sum_{k=-\infty}^{\infty} P(u + \frac{k}{T}) e^{j2\pi funT} du$$

$$= \int_{-1/2T}^{1/2T} T e^{j2\pi funT} du = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

Pulse Transmission System

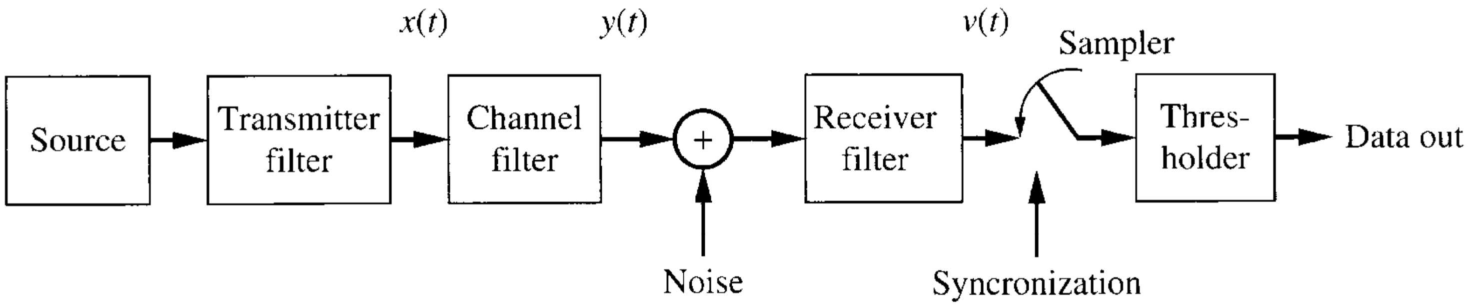


Figure 4.9
Transmitter, channel, and receiver cascade illustrating the implementation of a zero-ISI communication system.

$$x(t) = \sum_{k=-\infty} a_k h_T(t-kT).$$

$$v(t) = y(t) * h_R(t) = [x(t) * h_C(t)] * h_R(t).$$

Consider the combined effect of three filters, we would like to have $v(t)$ to be ISI-free.

Transmitter and Receiver Filters

Let $v(t) = A\sum_{k=-\infty}^{\infty} a_k p_{RC}(t-kT-t_d)$

where A represents a scale factor and t_d a possible delay.

$Ap_{RC}(t-t_d) = h_T(t) * h_C(t) * h_R(t)$, or in frequency domain,

$AP_{RC}(f)e^{-j2\pi f t_d} = H_T(f)H_C(f)H_R(f)$.

If $H_C(f)$ is given, then $|H_T(f)||H_R(f)| = 1/|H_C(f)|$

The overall spectrum should satisfy the Nyquist criterion to avoid ISI. $\Rightarrow$ A filter design problem.

- Practically, $H_c(f)$ is unknown of time-varying

Let $\left|H_T(f)\right| = \left|H_R(f)\right| = 1 \Big/ \left|H_C(f)\right|^{1/2}$

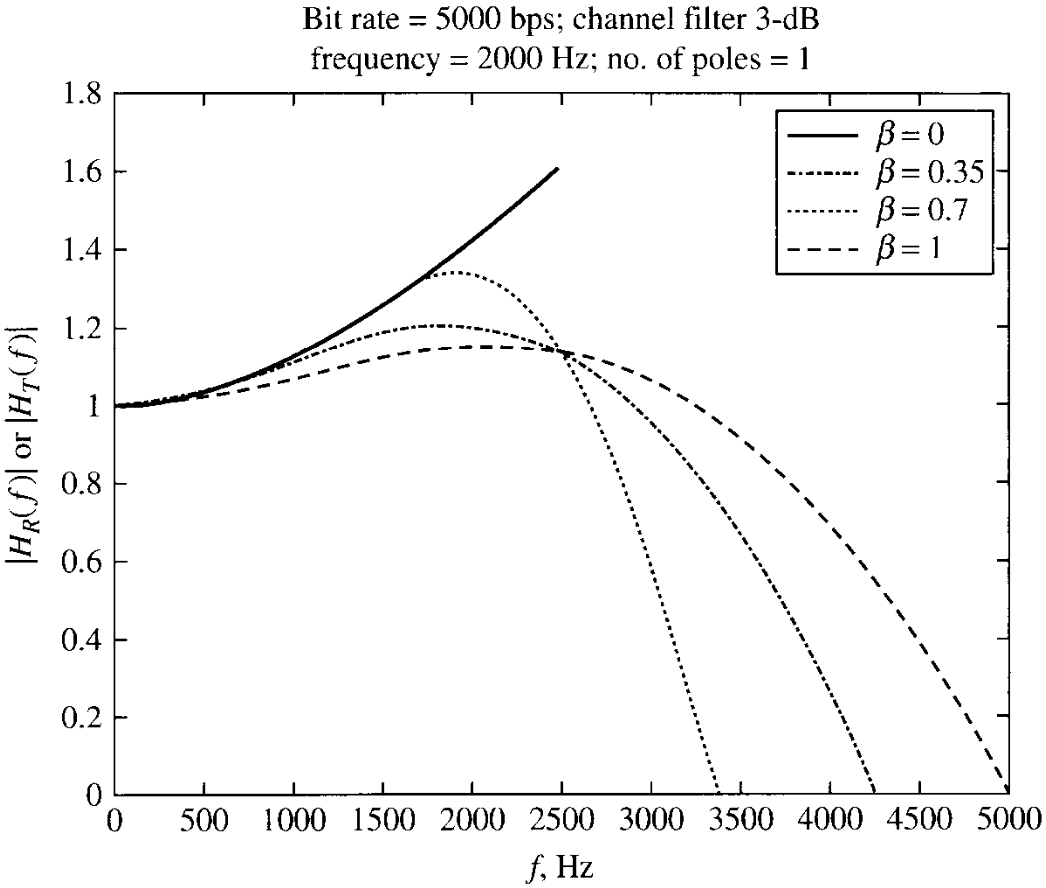


Figure 4.10

Transmitter and receiver filter amplitude responses that implement the zero-ISI condition assuming a first-order Butterworth channel filter and raised cosine pulse shapes.

Zero-Forcing Equalization

- Now, assume digital processing at the receiver.
- *Purpose*: Design an FIR filter that compensates for the channel distortion → zero-ISI.
- Show an example of zero-ISI equalizer design.

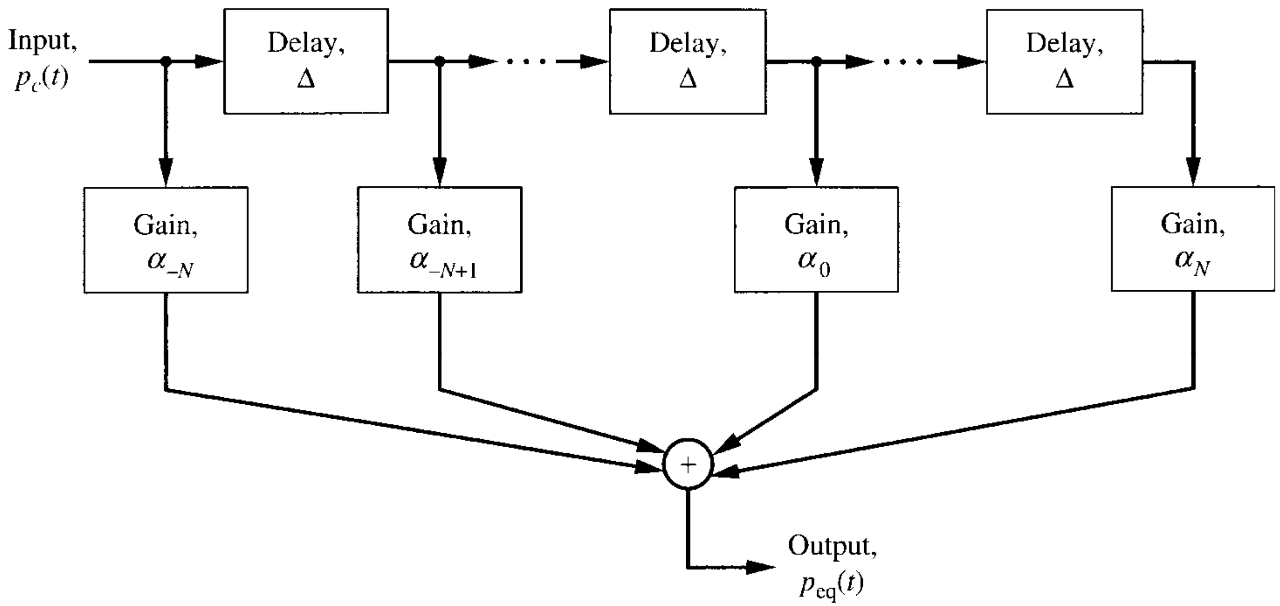


Figure 4.11
A transversal filter implementation for equalization of ISI.

Let the combined impulse response after the channel be $p_c(t)$, Pass such a pulse through our finite-length equalizer, then

$$p_{eq}(t) = \sum_{n=-N}^{n} \alpha_n p_c(t - n\Delta).$$

If the resulting pulse satisfies the zero-ISI condition, ISI-free transmission is achieved. Assume that $\Delta = T$, the condistion is

$$p_{eq}(mT) = \sum_{n=-N}^{n} \alpha_n p_c[(m-n)T] = \begin{cases} 1, & m = 0 \\ 0, & m \neq 0 \end{cases} \quad m = 0, \pm 1, \ldots, \pm N.$$

Notice that we only enforce the zero-ISI condition on the $2N+1$ samples. Why? We only have $2N+1$ coefficients (variables) and can only achieve this much. Solve the $2N+1$ equations and you obtain the zero-forcing equalizer.

$$\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} p_c(0) & p_c(-T) & \cdots & p_c(-2NT) \\ p_c(T) & p_c(0) & \cdots & p_c((-2N+1)T) \\ \vdots & & & \vdots \\ p_c(2NT) & & & p_c(0) \end{bmatrix} \cdot \begin{bmatrix} a_{-N} \\ a_{-N+1} \\ \vdots \\ a_N \end{bmatrix}$$

<u>Example</u>:

A 3-tap equalizer

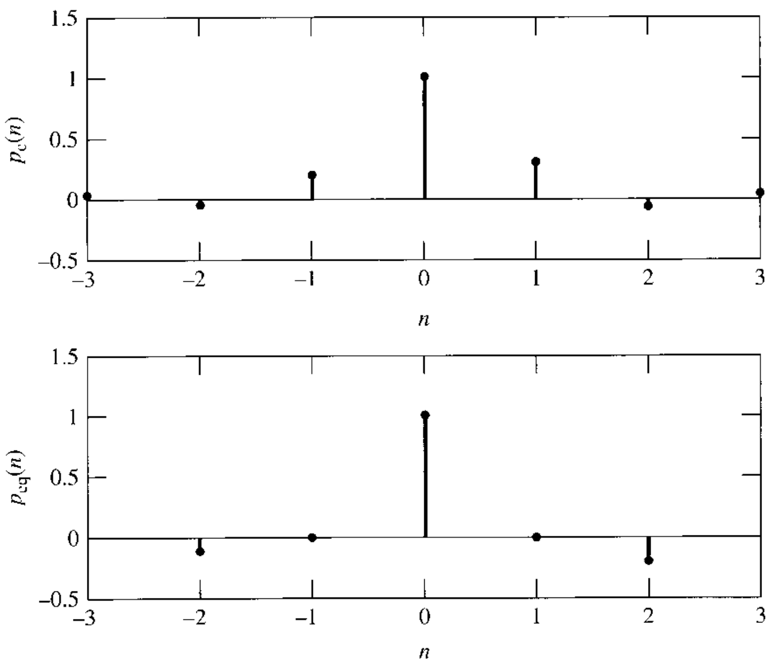


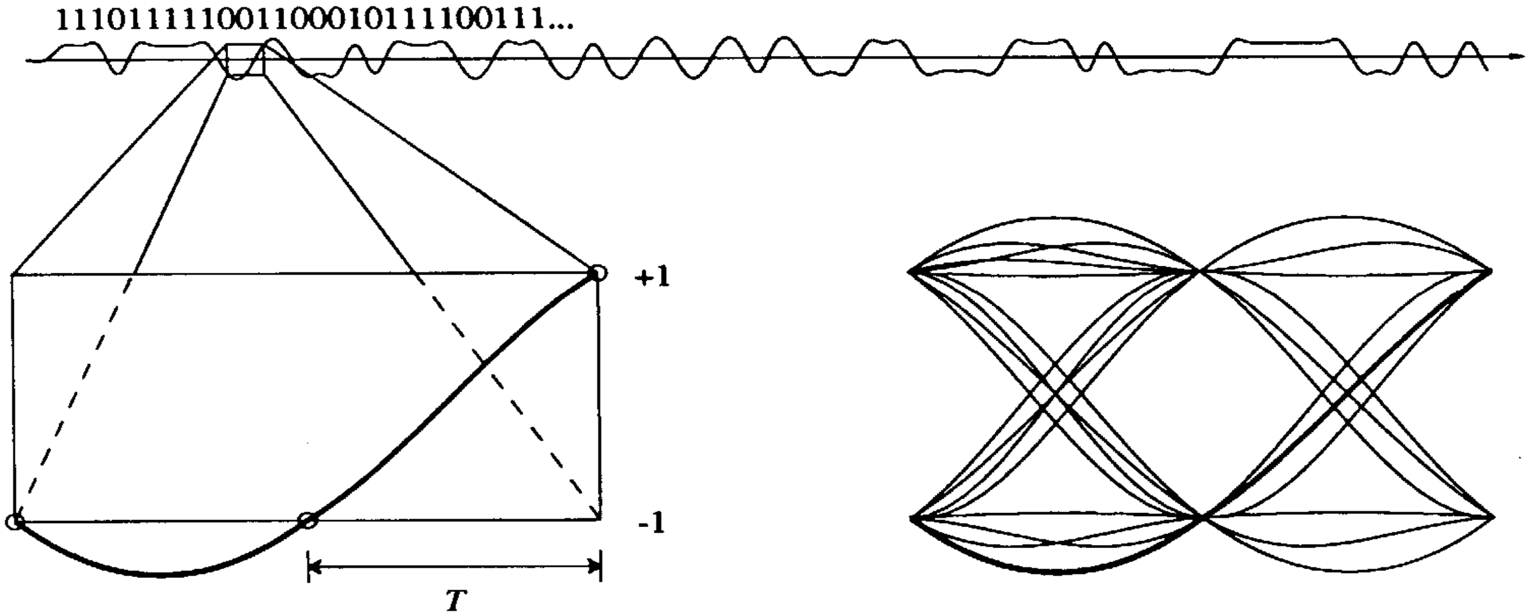
Figure 4.12
Samples for (a) an assumed channel response and for (b) the output of a zero-forcing equalizer of length 3.

The equalizer cannot eliminate ISI beyond its span.

Eye Diagrams

- **Eye diagram**: Constructed by overlapping *a number of* segments of the base-band signals
- A qualitative measure of the system performance.

(Lee et al., Digital Comm., 1994)



Increasing bandwidth may mitigate ICI, but will also allows more noise to enter. Another example of trade-offs in commu systems design

Amplitude jitter:
ICI causes the amplitude of each symbol fluctuate.

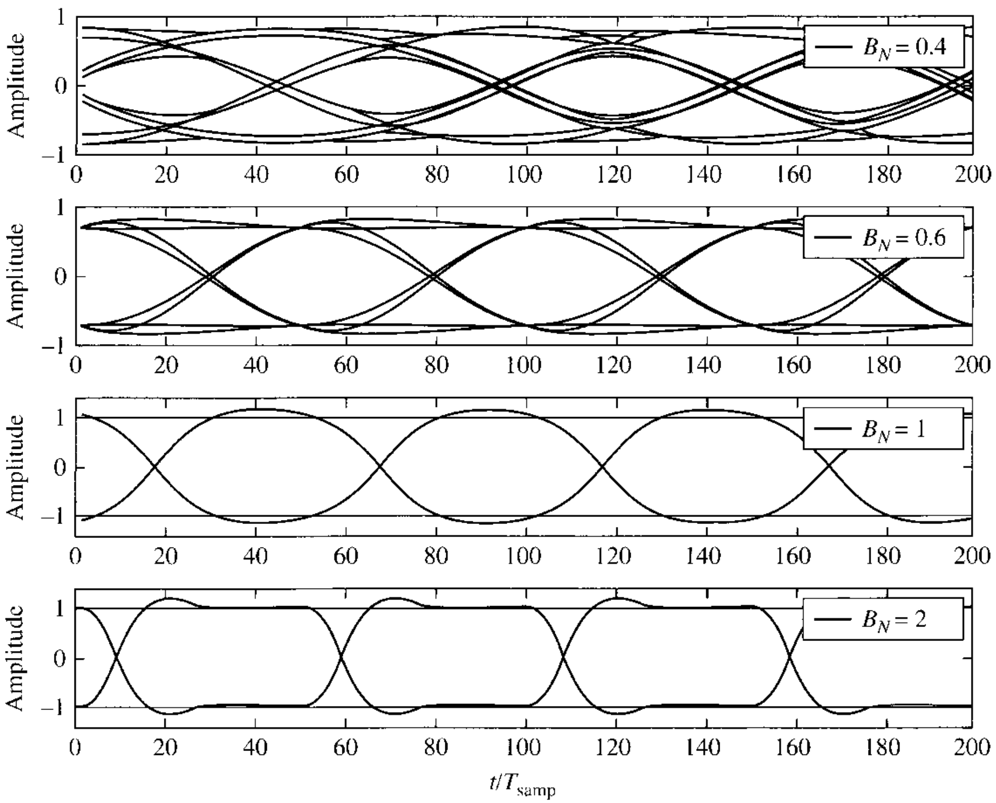


Figure 4.14
Eye diagrams for $B_N = 0.4, 0.6, 1.0,$ and 2.0.

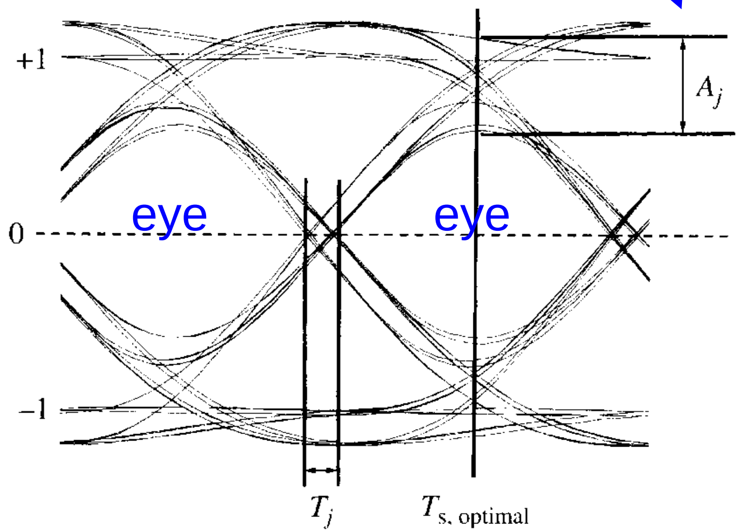


Figure 4.15
Two-symbol eye diagrams for $B_N = 0.4$.

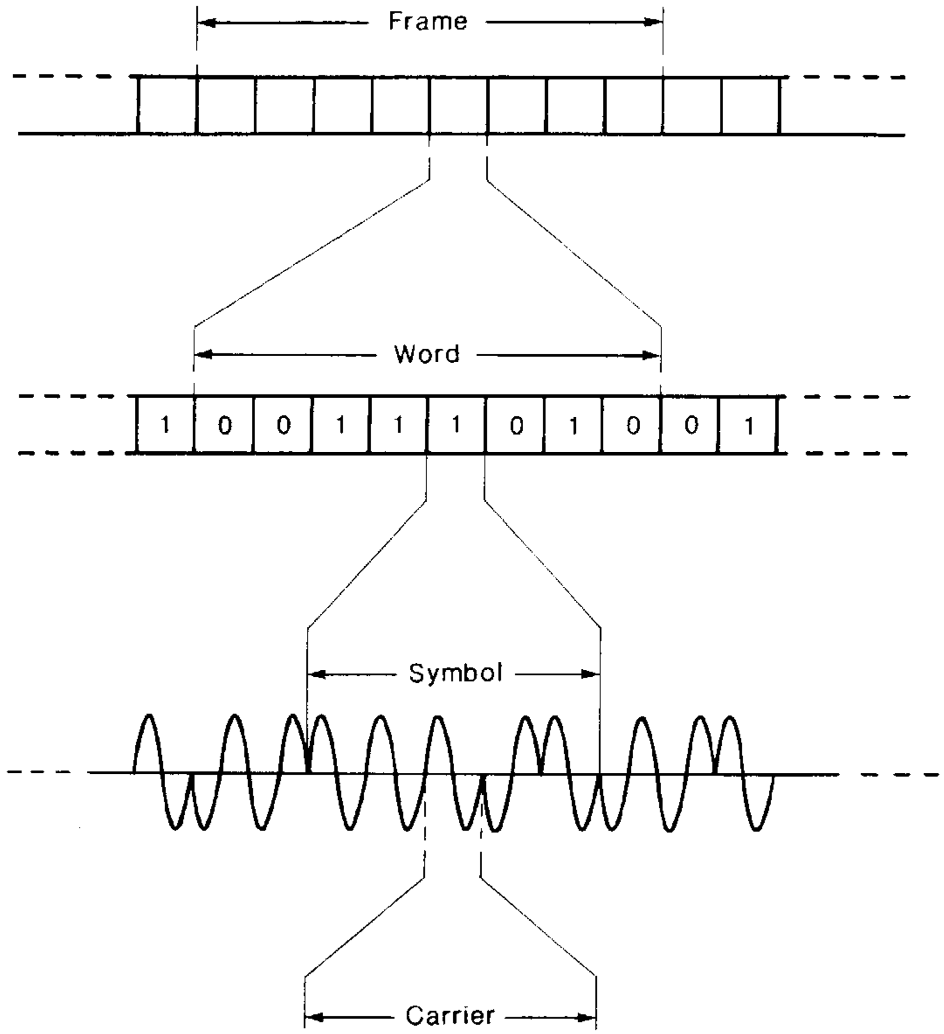
Timing jitter:
ICI causes the timing of each symbol fluctuate.

The best place to sample the signal.

Symbol, Bit, Word, and Frame

- Bits → Symbol (a single pulse in transmission) --- carrier sync., symbol timing
- Bits → Word --- word sync.
- Words → Frame --- frame sync.

(Benedetto et al., Digital Tansmission Theory, 1987)



Synchronization

- Methods : (1) external sync signals;

  (2) **self-synchronization**: derivation from the modulated signals

- **Example 1**: squaring the received NRZ signals and PLL

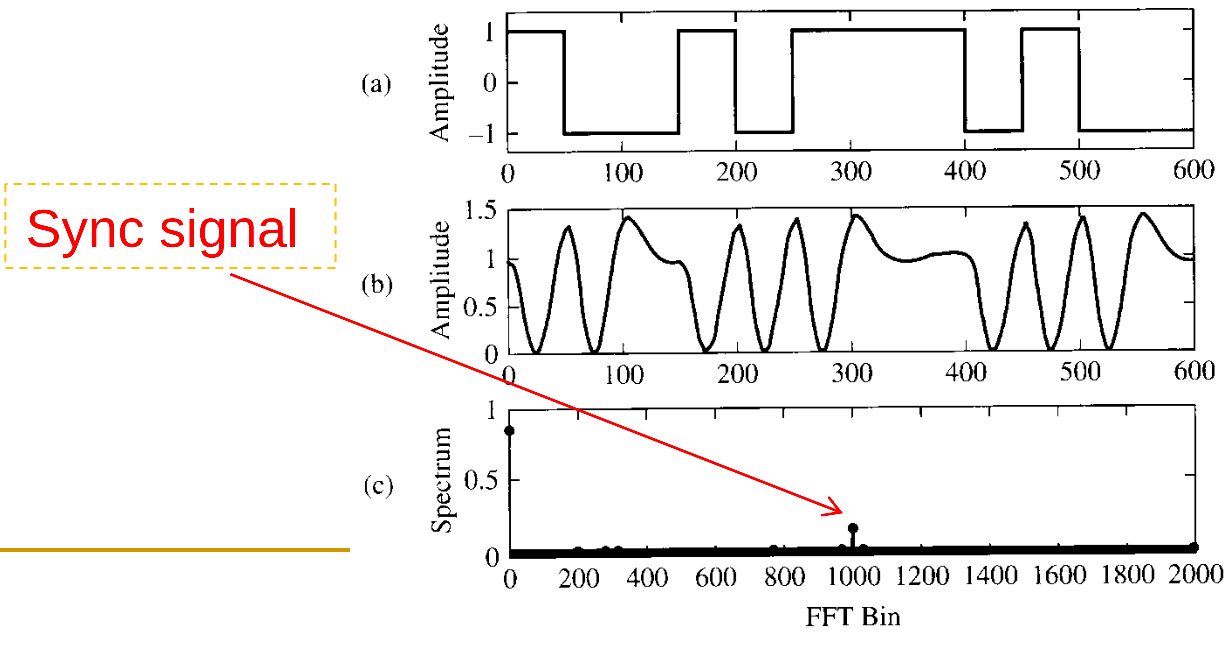


Figure 4.16
Simulation results for Computer Example 4.3. (a) NRZ waveform. (b) NRZ waveform filtered and squared. (c) FFT of squared NRZ waveform.

Example 2

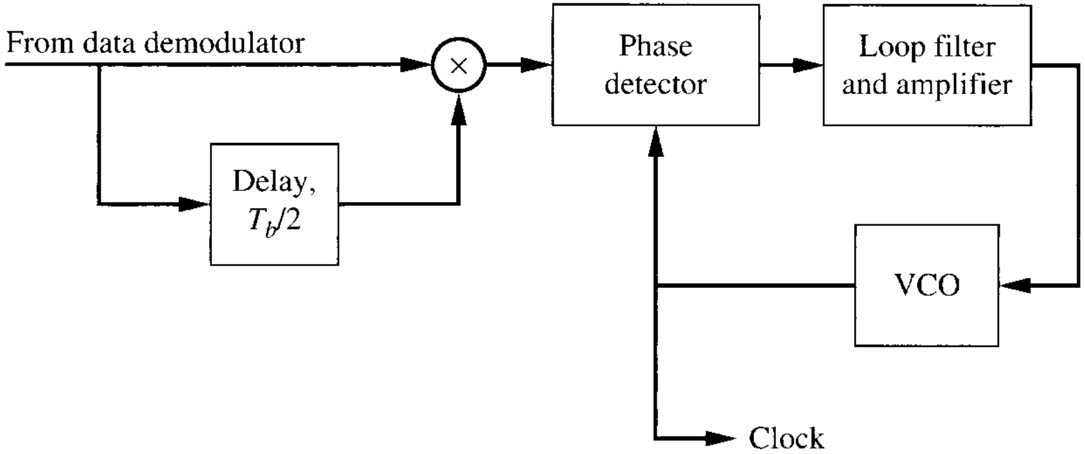


Figure 4.17
System for deriving a symbol clock simulated in Computer Example 4.~~3~~. 4

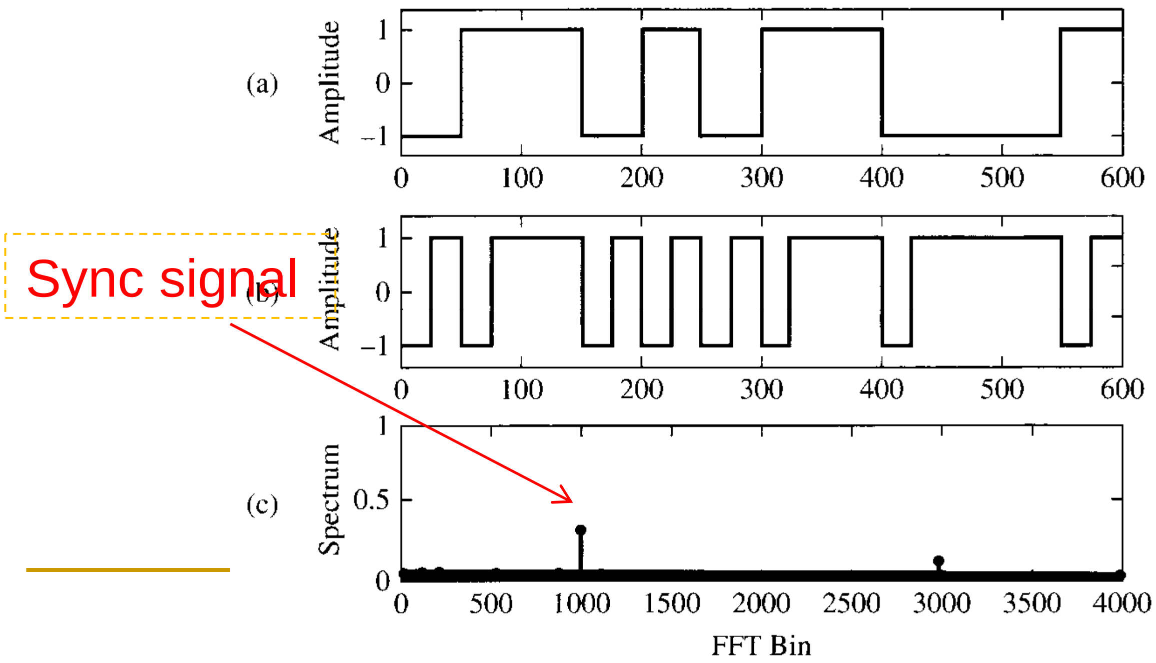


Figure 4.18
Simulation results for Computer Example 4.4. (a) Data waveform. (b) Data waveform multiplied by a half-bit delayed version of itself. (c) FFT spectrum of (b).

Carrier Modulation

- Simple modulation applied to baseband digital signals

Example: Let $d(t)$ be the NZR waveform.

Amplitude Shift-Keying (ASK): $x_{ASK}(t) = A_c[1 + d(t)]\cos(2\pi f_c t)$

Phase Shift-Keying (PSK): $x_{PSK}(t) = A_c \cos[2\pi f_c t + \frac{\pi}{2} d(t)]$

Frequency Shift-Keying (FSK): $x_{FSK}(t) = A_c \cos[2\pi f_c t + k_f \int^t d(\alpha) d\alpha]$

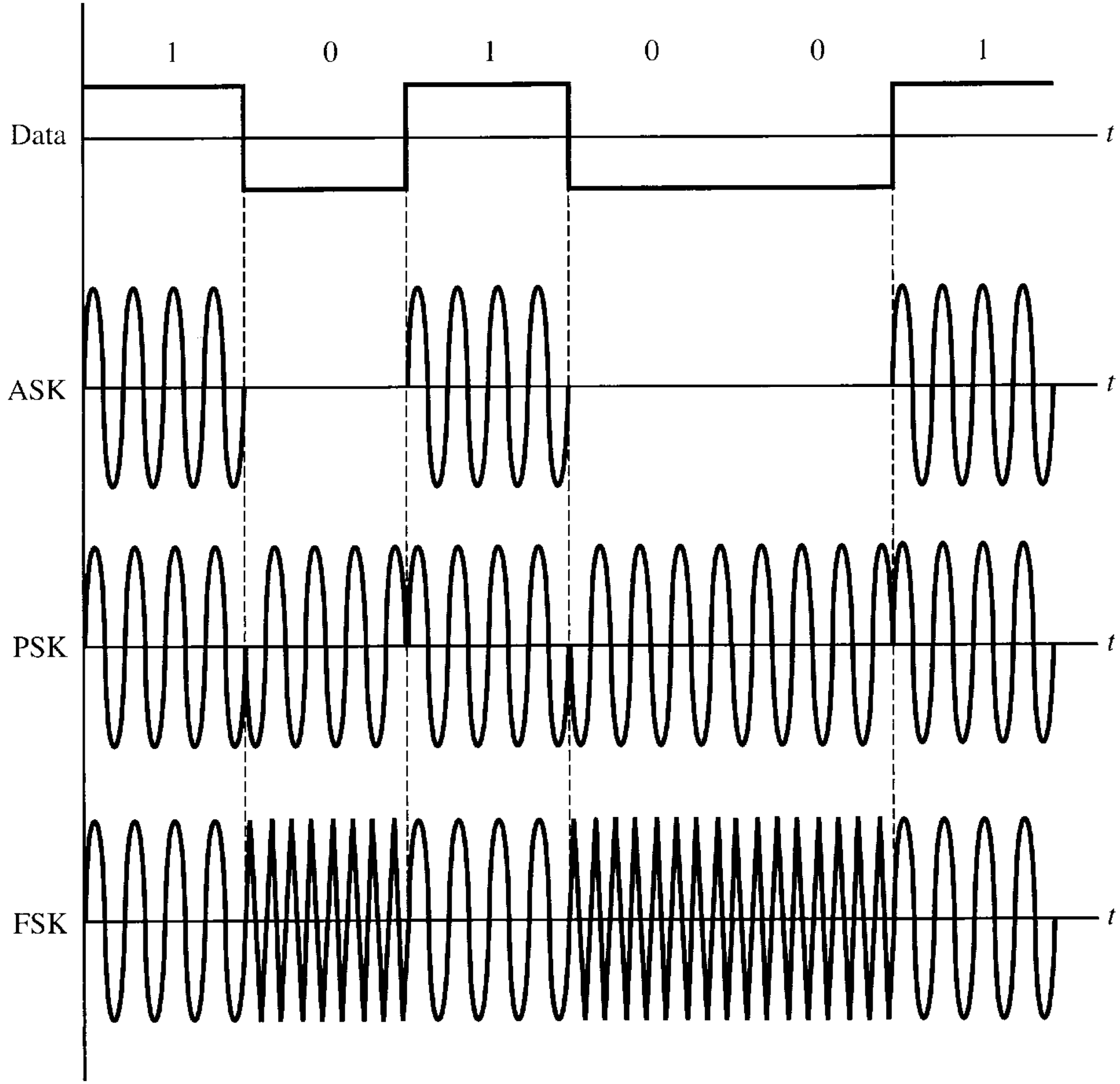


Figure 4.19
Examples of digital modulation schemes.