Principles of Communications Lecture 6: Analog Modulation Techniques (4)

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Outlines

- Linear Modulation
- Angle Modulation
- Interference
- Feedback Demodulators
- Analog Pulse Modulation
- Delta Modulation and PCM
- Multiplexing

Interference

- Interference: The undesired signal in demodulation
- Types: *deterministic*; *random*.
- Simple example: Assume a *single tone* interference:

  $A_i \cos(\omega_c + \omega_i)$; ω_c: carrier, ω_i: offset
- Analysis: interference in linear/angle mod

Interference in Linear Modulation

$$\begin{cases} \text{Message}: m(t) = \dfrac{A_m}{A_c}\cos\omega_m t \\ \text{Interference}: A_i\cos(\omega_c + \omega_i)t \end{cases}$$

- Received signal:

$$x_r(t) = A_C\cos\omega_c t + A_m\cos\omega_m t\cdot\cos\omega_c t + \underline{A_i\cos(\omega_c + \omega_i)t}$$

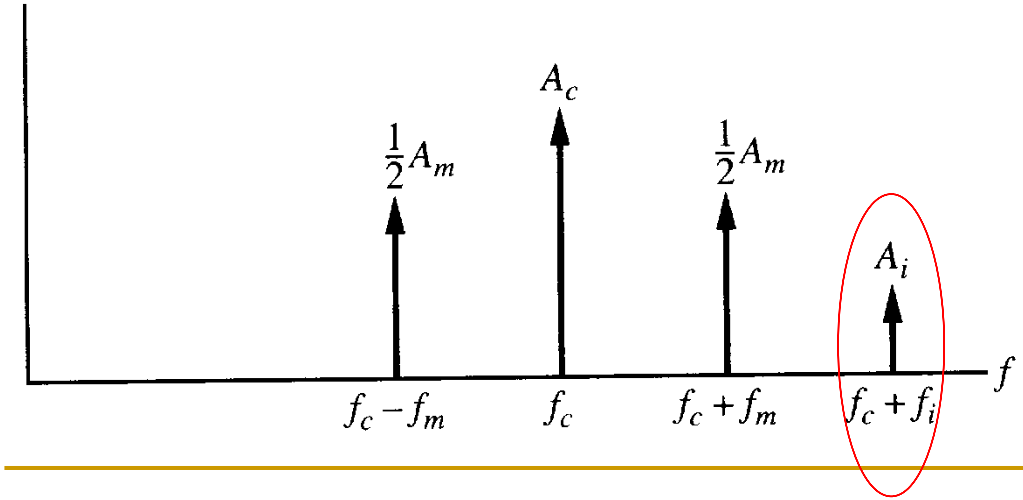


Figure 3.38
Assumed received-signal spectrum.

Receiver Output

1. Coherent detection: *Linear* detector

$$y_D(t) = A_m \cos\omega_m t + A_i \cos\omega_i t$$

Additive; no increase, no decrease

2. Envelope detection: *Nonlinear* detector

$$x_r(t) = \mathrm{Re}\{e^{j\omega_c t}[A_C + A_i e^{j\omega_1 t} + \frac{1}{2}A_m e^{j\omega_m t} + \frac{1}{2}A_m e^{-j\omega_m t}]\}$$

$\downarrow$ One-sided spectrum → phasor

$$\begin{aligned} x_r(t) &= A_C \cos\omega_c t + A_m \cos\omega_m t \cdot \cos\omega_c t + A_i \cos(\omega_c + \omega_i)t \\ &= A_C \cos\omega_c t + A_m \cos\omega_m t \cdot \cos\omega_c t + A_i[\cos\omega_c t \cos\omega_i t - \sin\omega_c t \sin\omega_i t] \\ &= [A_C + A_m \cos\omega_m t + A_i \cos\omega_i t]\cdot\cos\omega_c t - A_i \sin\omega_i t \sin\omega_c t \end{aligned}$$

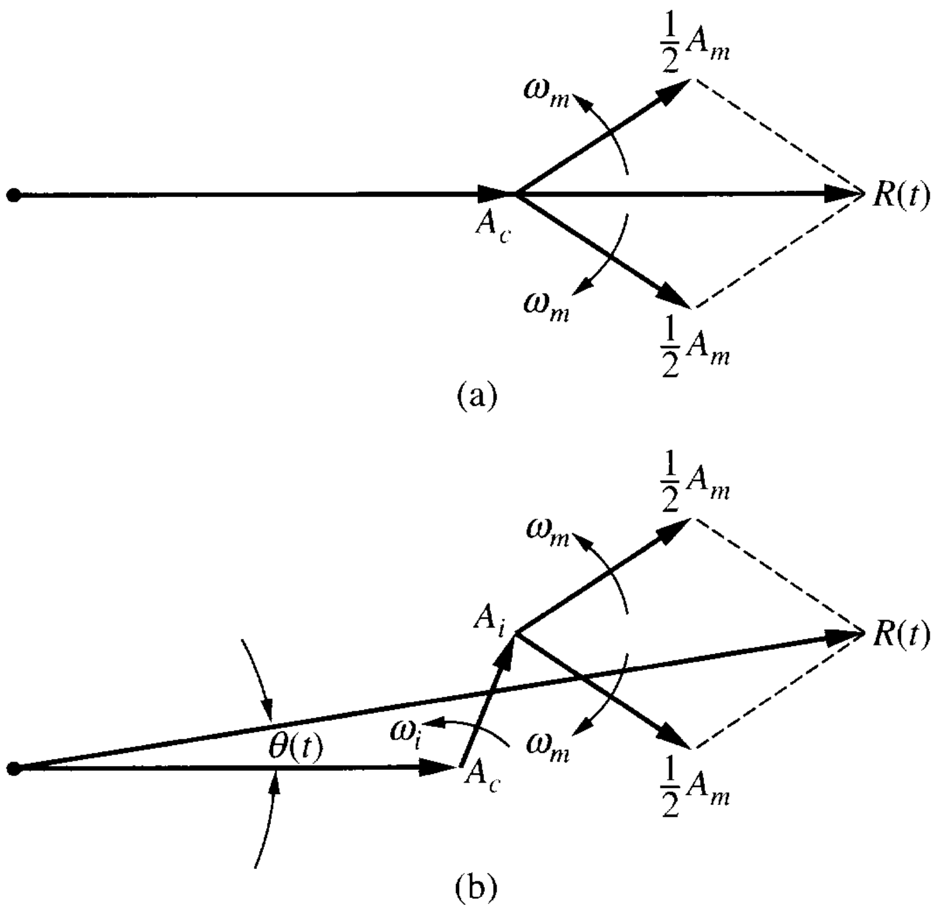


Figure 3.39
Phasor diagrams illustrating interference.
(a) Phasor diagram without interference.
(b) Phasor diagram with interference.

- Case (i) If $A_C >> A_i$ (typical case)

$$x_r(t) = [A_C + A_m \cos\omega_m t + A_i \cos\omega_i t] \cdot \cos\omega_c t$$

Envelope of $x_r(t) = A_C + A_m \cos\omega_m t + A_i \cos\omega_i t$

$y_D(t) = A_m \cos\omega_m t + A_i \cos\omega_i t$ (same as the coherent detector)

- Case (ii) If $A_C << A_i$

$$\begin{aligned}
x_r(t) &= A_C \cos(\omega_c + \omega_i - \omega_i)t + A_m \cos\omega_m t \cdot \cos(\omega_c + \omega_i - \omega_i)t + A_i \cos(\omega_c + \omega_i)t \\
&= A_C[\cos(\omega_c + \omega_i)t \cdot \cos\omega_i t + \sin(\omega_c + \omega_i)t \cdot \sin\omega_i t] + A_i \cos(\omega_c + \omega_i)t \\
&\quad + A_m \cos\omega_m t \cdot [\cos(\omega_c + \omega_i)t \cdot \cos\omega_i t + \sin(\omega_c + \omega_i)t \cdot \sin\omega_i t] \\
&= [A_i + A_C \cos\omega_i t + A_m \cos\omega_m t \cos\omega_i t] \cdot \cos(\omega_c + \omega_i)t \\
&\quad + [A_C \sin\omega_i t + A_m \cos\omega_m t \sin\omega_i t] \cdot \sin(\omega_c + \omega_i)t \\
&\approx [A_i + A_C \cos\omega_i t + A_m \cos\omega_m t \cos\omega_i t] \cdot \cos(\omega_c + \omega_i)t
\end{aligned}$$

$A_c\sin + A_m\cos << A_i$

Envelope of $x_r(t) \approx A_i + A_C \cos\omega_i t + A_m \cos\omega_m t \cos\omega_i t$

$y_D(t) = A_C \cos\omega_i t + A_m \cos\omega_m t \cos\omega_i t$

Where is the message? Lost! (Nonlinear)
This is called **threshold effect**.

Threshold Effect

- When the interference is greater than a certain level, the message is totally lost. It is a *non-linear* phenomenon.

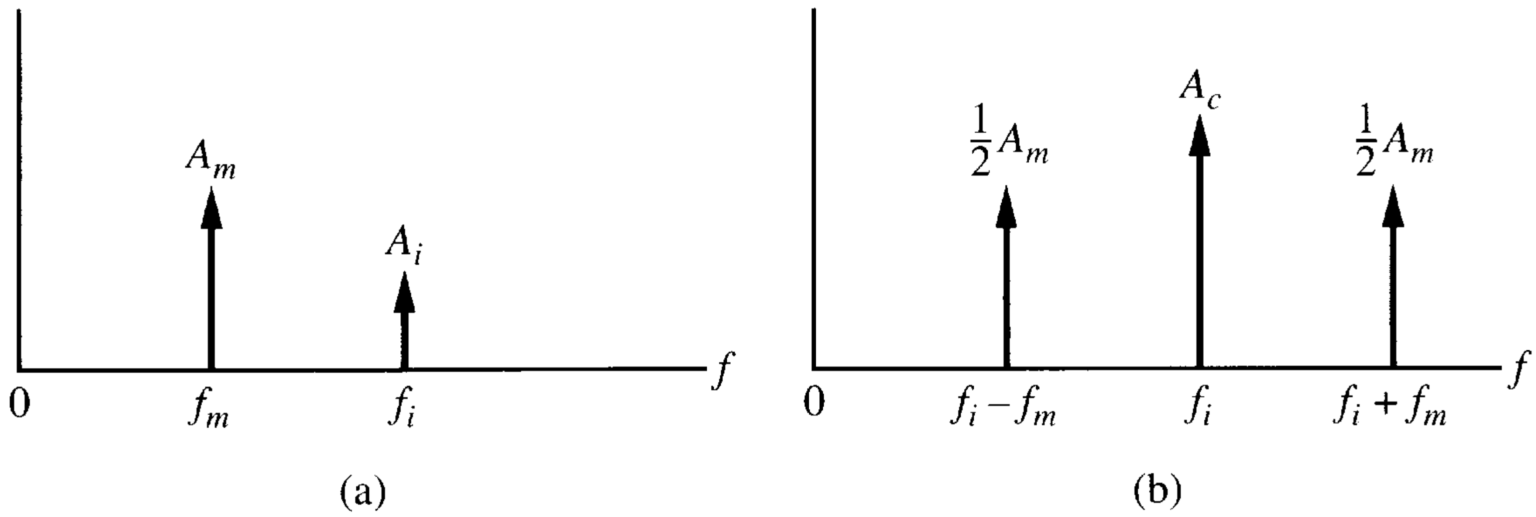


Figure 3.40
Envelope detector output spectra. (a) $A_c \gg A_i$. (b) $A_c \ll A_i$.

Interference in Angle Modulation

- No message assumed, but with interference at $(\omega_c + \omega_i)$.

$$x_r(t) = A_C \cos(\omega_c t + \cancel{\phi}(t)) + A_i \cos(\omega_c + \omega_i)t$$

$$= A_C \cos\omega_c t + A_i \cos\omega_i t \cos\omega_c t - A_i \sin\omega_i t \sin\omega_c t$$

$$= [A_c + A_i \cos\omega_i t]\cos\omega_c t - [A_i \sin\omega_i t]\sin\omega_c t$$

$$= R(t)\cos[\omega_c t + \psi(t)]$$

$$\text{where } \begin{cases} R(t) = \sqrt{(A_C + A_i\cos\omega_i t)^2 + (A_i \sin\omega_i t)^2} \\ \psi(t) = \tan^{-1}\left(\dfrac{A_i \sin\omega_i t}{A_C + A_i\cos\omega_i t}\right) \end{cases}.$$

- How does the interference behave after demodulation?

- Case (i) If $A_C >> A_i$

$$\begin{cases} R(t) \approx A_C + A_i \cos \omega_i t \\ \varphi(t) \approx \tan^{-1}\left(\dfrac{A_i \sin \omega_i t}{A_C}\right) \approx \dfrac{A_i \sin \omega_i t}{A_C} \end{cases}$$

$$\boxed{\tan^{-1} z = z - \tfrac{1}{3} z^3 + \tfrac{2}{15} z^5 \cdots}$$

Assume an ideal discriminator extracts $\phi(t), \dfrac{d\phi(t)}{dt}$

$$\begin{cases} \text{For PM:} \quad y_D(t) = K_D \dfrac{A_i}{A_C} \sin \omega_i t \\ \text{For FM:} \quad y_D(t) = \dfrac{1}{2\pi} K_D \dfrac{d}{dt}\left(\dfrac{A_i}{A_C} \sin \omega_i t\right) = K_D \dfrac{A_i}{A_C} f_i \cos \omega_i t \end{cases}$$

- When f_i is small, interference in FM becomes ***smaller***.
- If $f_i > W$ (message BW), interference is filtered out.

Small Interference

- In summary, for $A_c >> A_i$, the single tone interference $A_i\cos(\omega_c+\omega_i)$ at the demodulator output:

$$
\begin{cases}
\text{AM:} & \dfrac{1}{a}\dfrac{A_i}{A_C}\cos\omega_i t \\[2ex]
\text{PM:} & K_D\dfrac{A_i}{A_C}\sin\omega_i t \\[2ex]
\text{FM:} & K_D\dfrac{A_i}{A_C}f_i\cos\omega_i t
\end{cases}
$$

- If they all have the same constants,

$$\frac{1}{a}\frac{A_i}{A_C} = K_D\frac{A_i}{A_C} = K_D\frac{A_i}{A_C}$$

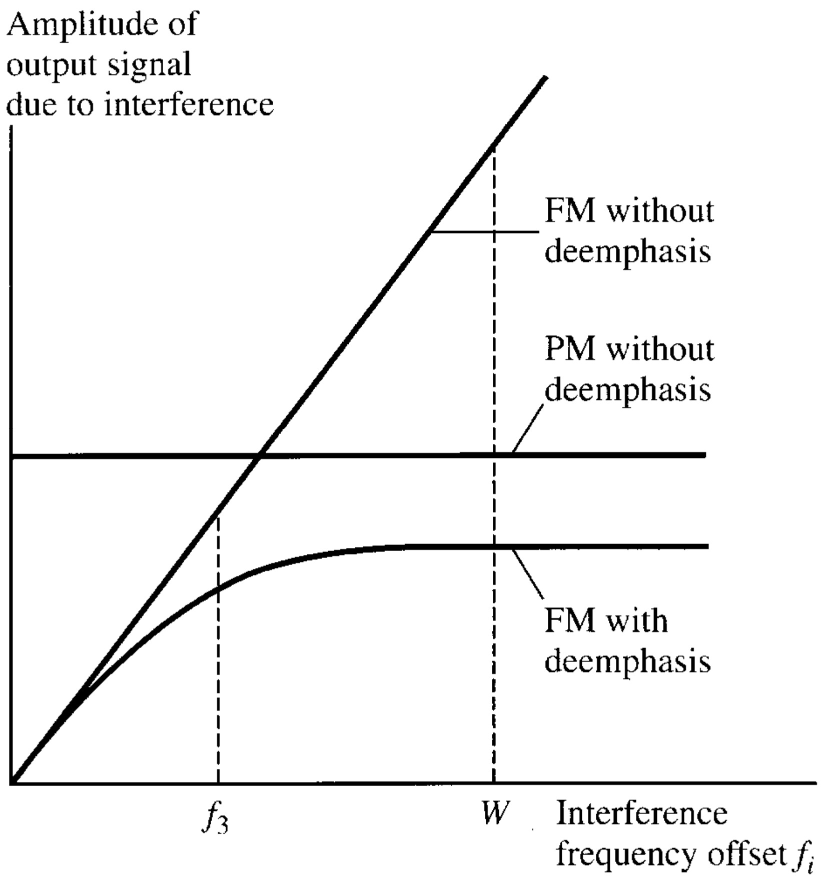
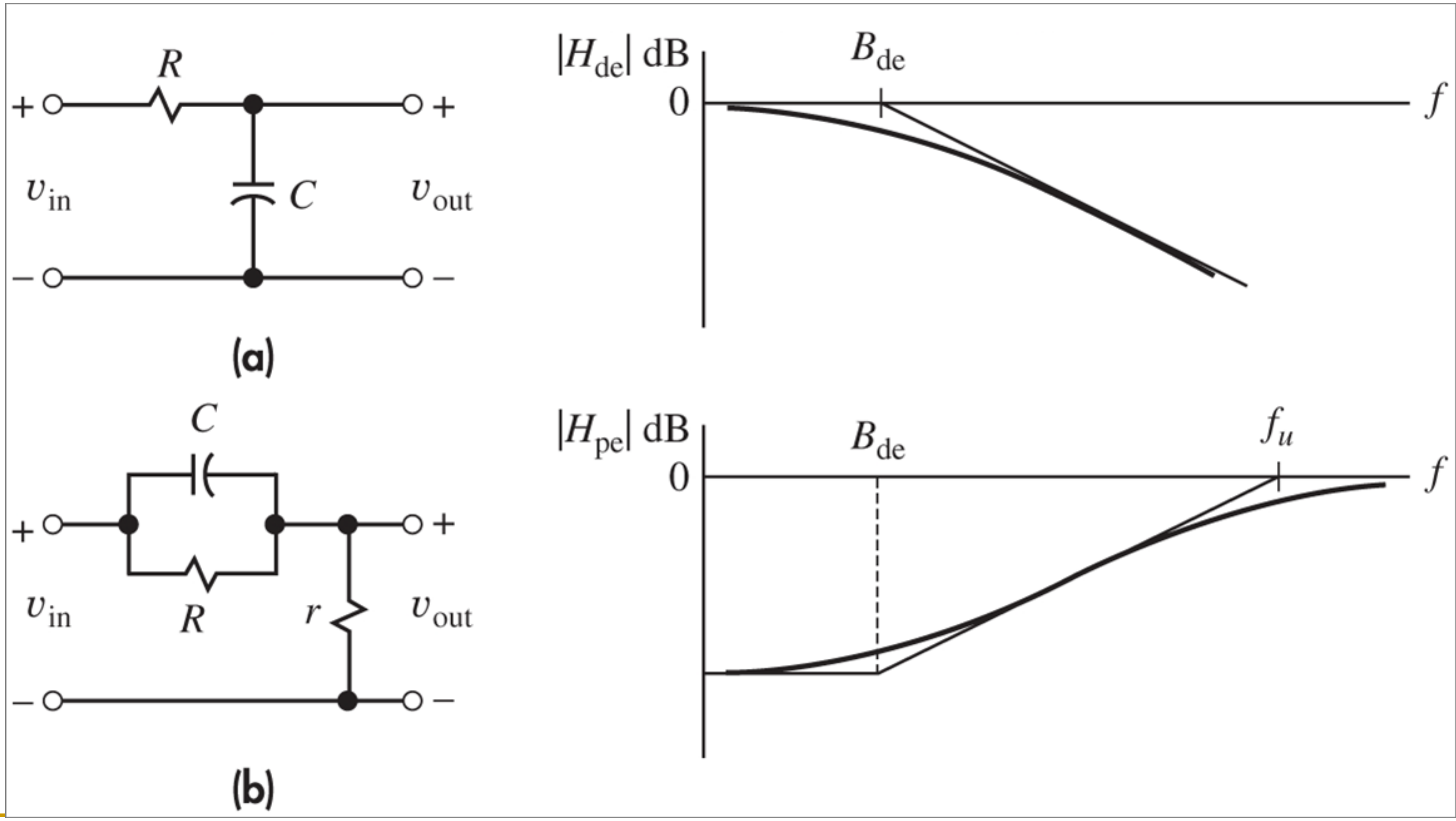


Figure 3.43
Amplitude of discriminator output due to interference.

FM Pre-emphasis/De-emphasis

- Amplify high freq components of message before modulation (Carlson, Fig.5-4.5)



- Case (ii) If $A_C < A_i$ (difficult to analyze)

Make a rough analysis based on phasor diagram.

$$x_r(t) = A_C \cos\omega_c t + A_i \cos(\omega_c + \omega_i)t$$

$$= \mathrm{Re}\{[A_C + A_i e^{j\omega_i t}]e^{j\omega_c t}\}$$

Interference phase $\theta(t) = \omega_i t$. Resultant phase $\psi(t)$.

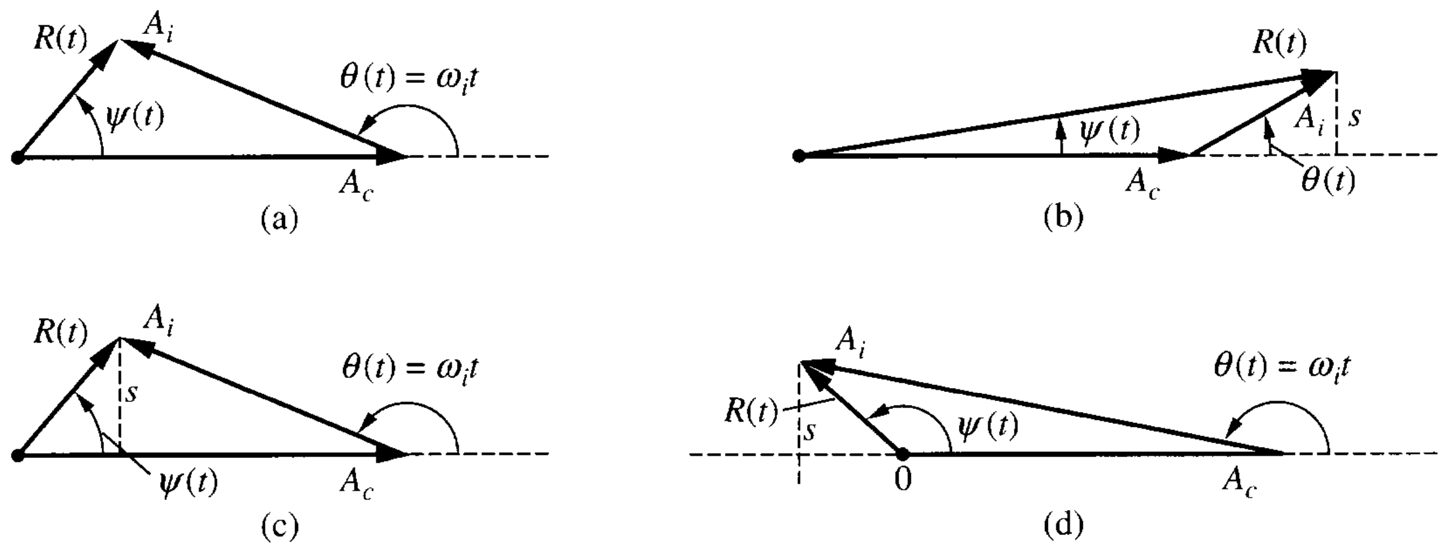


Figure 3.41

Phasor diagram for carrier plus single-tone interference. (a) Phasor diagram for general $\theta(t)$. (b) Phasor diagram for $\theta(t) \approx 0$. (c) Phasor diagram for $\theta(t) \approx \pi$ and $A_i \lesssim A_c$. (d) Phasor diagram for $\theta(t) \approx \pi$ and $A_i \gtrsim A_c$.

(1) $\theta(t) \approx 0$

$$s \cong \theta(t) \cdot A_i \qquad (\sin\theta(t) \cdot A_i)$$

$$\approx (A_C + A_i) \cdot \psi(t)$$

$$\psi(t) \approx \frac{A_i}{A_C + A_i} \omega_i t$$

$$\boxed{y_D(t) = \frac{K_D}{2\pi} \frac{d\psi(t)}{dt} = K_D \frac{A_i}{A_C + A_i} f_i}$$

(2) $\begin{cases} \theta(t) \approx \pi \\ A_i <\approx A_C \end{cases}$

$$s \cong (\pi - \theta(t)) \cdot A_i \approx (A_C - A_i) \cdot \psi(t)$$

$$\psi(t) \approx \frac{A_i(\pi - \omega_i t)}{A_C - A_i}$$

$$\boxed{y_D(t) = -K_D \frac{A_i}{A_C - A_i} f_i}$$

(3) $\begin{cases} \theta(t) \approx \pi \\ A_i >\approx A_C \end{cases}$

$$s \cong (\pi - \theta(t)) \cdot A_i \approx (A_i - A_C) \cdot (\pi - \psi(t))$$

$$\psi(t) \approx \pi + \frac{A_i(\pi - \omega_i t)}{A_i - A_C}$$

$$\boxed{y_D(t) = K_D \frac{A_i}{A_i - A_C} f_i}$$

$\Rightarrow$ We do not want to operate in the environment, where A_c $<\approx A_i$.

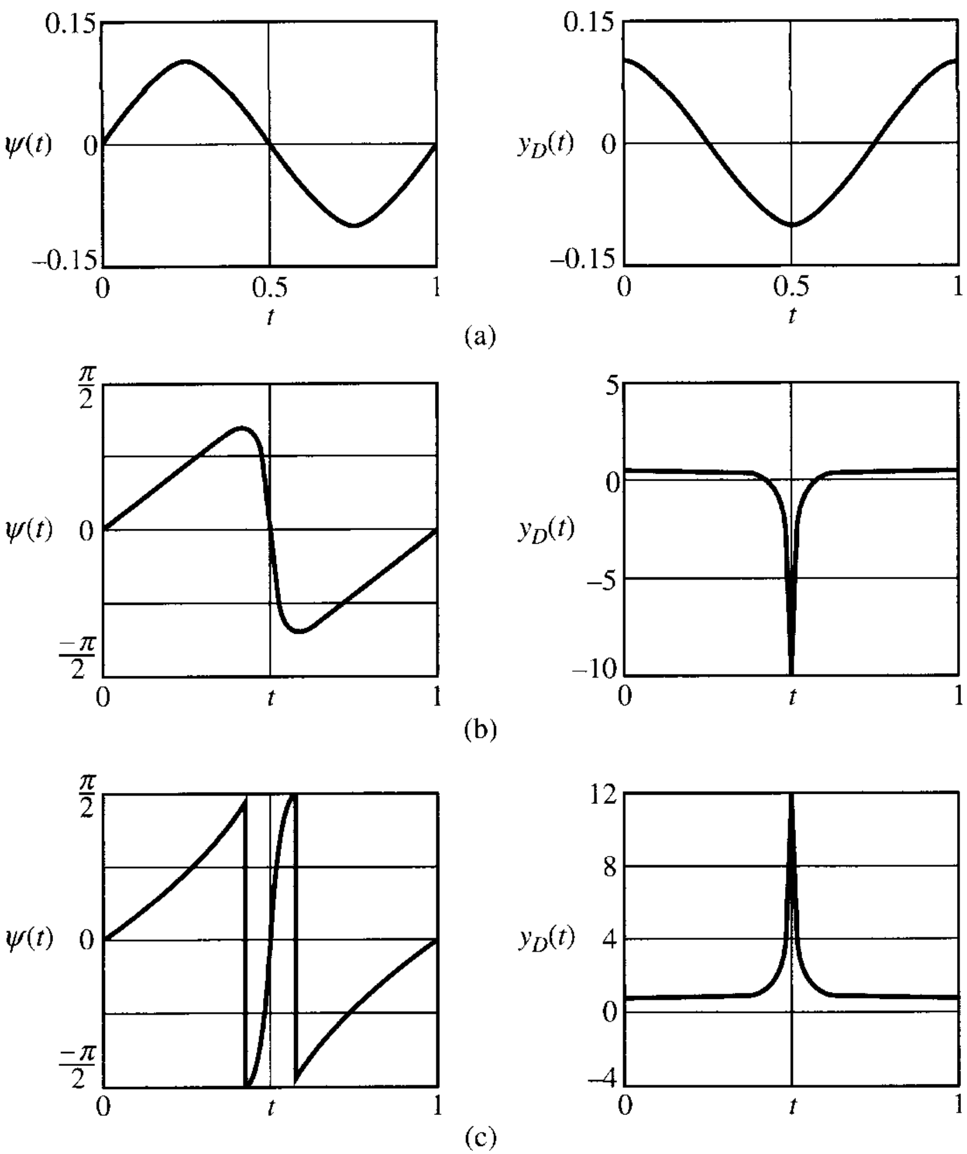


Figure 3.42

Phase deviation and discriminator outputs due to interference. (a) Phase deviation and discriminator output for $A_i = 0.1A_c$. (b) Phase deviation and discriminator output for $A_i = 0.9A_c$. (c) Phase deviation and discriminator output for $A_i = 1.1A_c$.

Phase-Lock Loops (PLL)

- **PLL for FM Demodulation** -- Tracks the instantaneous angle (phase and frequency) of the input signal.
  (1) phase detector (comparator)
  (2) loop filter
  (3) loop amplifier
  (4) VCO (voltage-controlled oscillator)

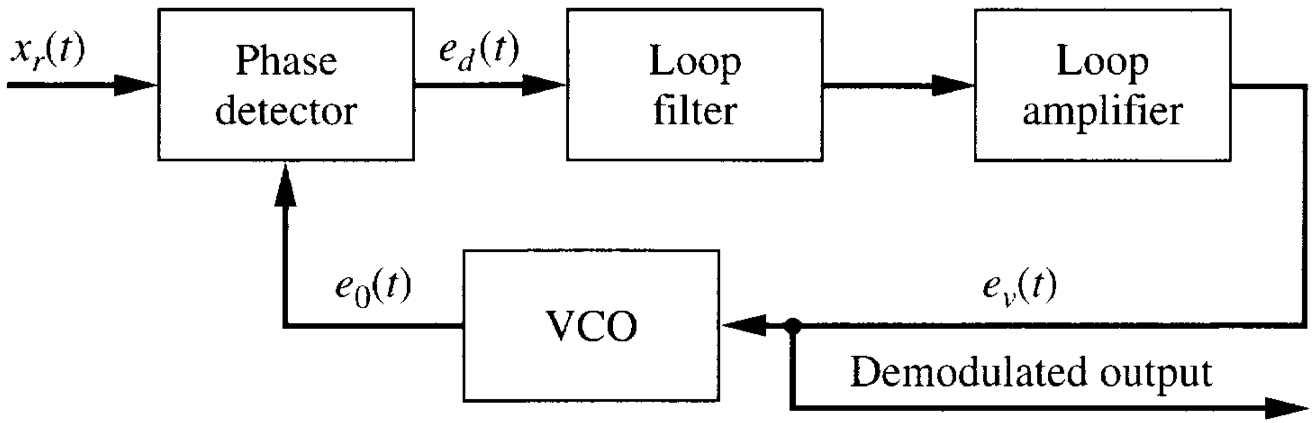


Figure 3.45
Phase-locked loop.

Principle

- Basic operation: Adjust the phase of the local VCO output ($e_o(t)$) to match the input ($x_r(t)$) signal phase.

Input: $x_r(t) = A_C \cos[\omega_c t + \phi(t)]$

VCO ouput: $e_o(t) = A_V \sin[\omega_c t + \theta(t)]$

Our goal: $\theta(t) \xrightarrow{matches} \phi(t)$

• Phase Detector: $e_d(t) = g(\phi(t) - \theta(t))$.

Function $g()$ is the characteristic function of the detector.

Ideal $g()$: sawtooth $= K_d(\phi(t) - \theta(t))$

Example: [Multiplier+LPF] $e_d(t) = \dfrac{A_C A_V K_d}{2} \sin(\phi(t) - \theta(t))$.

$$e_V(t) \longrightarrow \boxed{\text{VCO}} \longrightarrow e_o(t) = A_V \sin[\omega_c t + \theta(t)]$$

VCO: An oscillator with frequency controlled by the input voltage

$$\frac{d\theta(t)}{dt} = K_V e_V(t) \quad \text{rad/s}$$

(K_V is the VCO constant)

$$\theta(t) = K_V \int^t e_V(\alpha) d\alpha.$$

- *Simplification*: Assume the frequency (ω_c) matches, only the phase ($\theta(t)$) needs to be adjusted.

General Model

- Assumption: the Loop filter is an LTI system, $F(s)$

$$E_v(s) = F(s)E_d(s)$$

$$\Leftrightarrow e_v(\alpha) = \int^t e_d(\lambda) f(\alpha - \lambda) d\lambda$$

$$\rightarrow \theta(t) = K_t \int^t \int^\alpha \sin[\phi(\lambda) - \theta(\lambda)] f(\alpha - \lambda) d\lambda d\alpha$$

$$\text{where } \; K_t = \tfrac{1}{2} A_v A_c K_d K_v$$

- Liner model:

$$\text{If } \phi(t) \approx \theta(t), \; \sin(\phi(t) - \theta(t)) \approx \phi(t) - \theta(t).$$

PLL Issues

- *Model*: Linear and Nonlinear
- Tracking Mode and Acquisition Mode
- *Key Parameters*:

  Lock range (frequency) & Lock time

  Pull-in range & Pull-in time

  Hold range

  Pull-out range

  (R. Best, *Phase-locked Loops*, 5th ed., McGraw-Hill)

Modes

- Tracking (locked) Mode:

  The PLL is already locked on to the signal – its phase (freq) is aligned with the incoming signal. Generally, the phase error is small and the *linear analysis* is used.

- Acquisition (unlocked) Mode:

  The PLL is either out of lock or just starts to lock on to a signal. This mode is *nonlinear* and is often difficult to analyze.

Parameters

- **Lock range**: This is the freq range within which a PLL locks within one single beat note. Normally, the operation freq range of a PLL is restricted to the lock range.
- **Pull-in range**: This is the freq range within which a PLL will always become locked, but the process can be rather slow.

- **Lock time**: The time PLL needs to get locked when the acquisition process is a (fast) lock-in process
- **Pull-in time**: The time PLL needs to get locked when the acquisition process is a (slow) pull-in process

Nonlinear Model

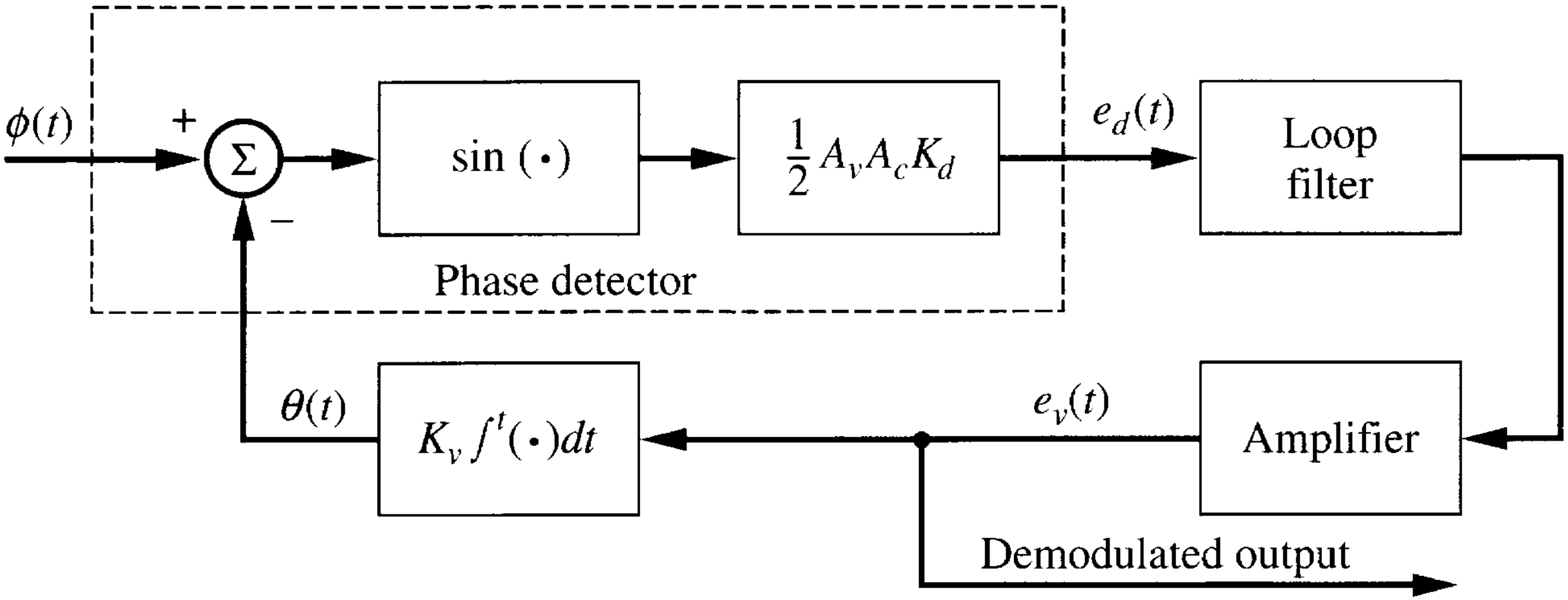


Figure 3.46
Nonlinear PLL model.

Linear Model

Accuracy: <4%
For |. |<0.5 rad

If $\phi(t) \approx \theta(t),\ \sin(\phi(t)-\theta(t)) \approx \phi(t)-\theta(t).$

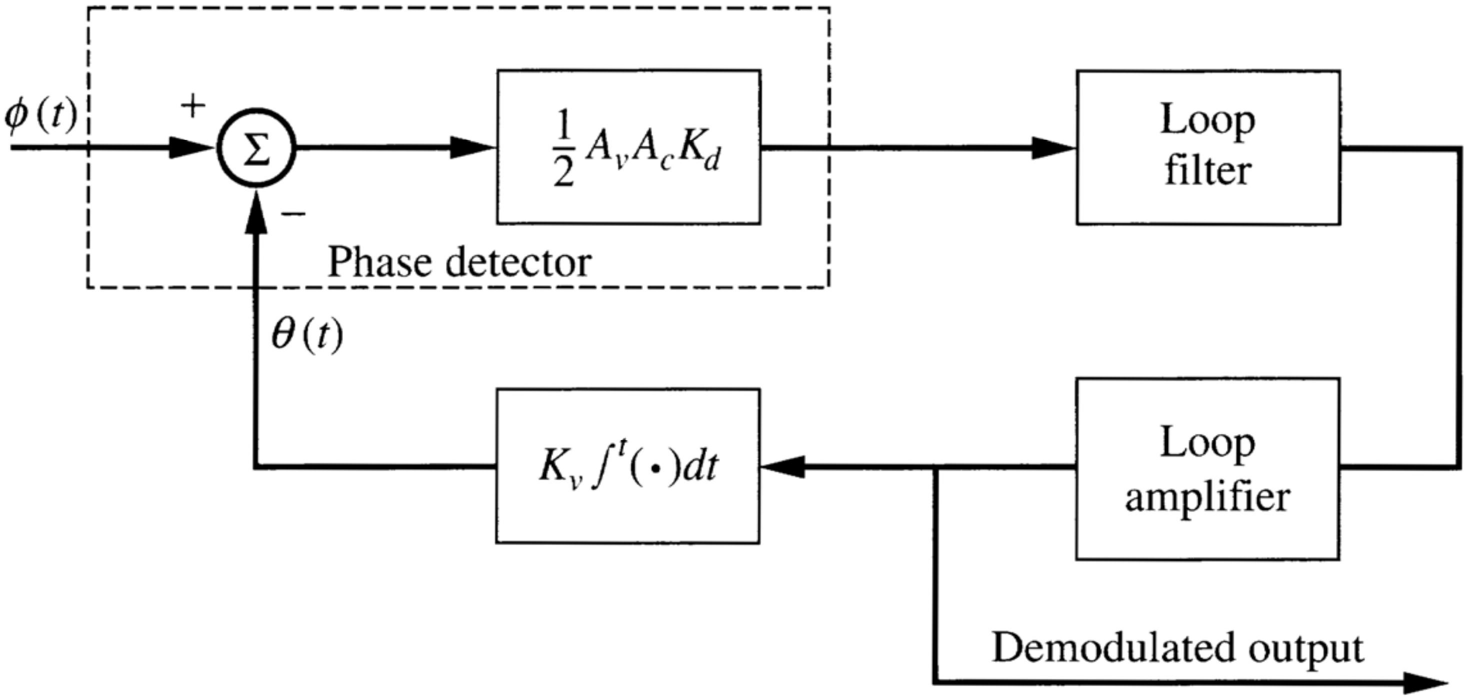


Figure 3.47
Linear PLL model.

Analysis on different loop filter assumption.

<Case 1> Loop filter = 1

(i) Nonlinear model

$$e_V(t) = \frac{\mu A_C A_V K_d}{2} \sin(\phi(t) - \theta(t))$$

and $\theta(t) = K_V \int e_V(\alpha) d\alpha = K_t \int^t \sin(\phi(\alpha) - \theta(\alpha)) d\alpha$

$$\frac{d\theta(t)}{dt} = K_t \sin(\phi(t) - \theta(t)).$$

Input, not under our control

<u>Ex:</u> Assume a *<u>frequency jump</u>* $\Delta\omega$ at input. (FM with a step function message)

Step Input

Let $\psi(t) = \phi(t) - \theta(t)$,

then $\dfrac{d\theta(t)}{dt} = \dfrac{d\phi(t)}{dt} - \dfrac{d\psi(t)}{dt} = \Delta\omega - \dfrac{d\psi(t)}{dt} = K_t \sin\psi(t)$

$$\Rightarrow \frac{d\psi(t)}{dt} + K_t \sin\psi(t) = \Delta\omega.$$

- The solution of $\phi(t)$ tells us this PLL behavior.
- Graphical representation (analysis): **Phase-plane plot** (diagram of $\phi(t)$ and $d\,\phi(t)/dt$)

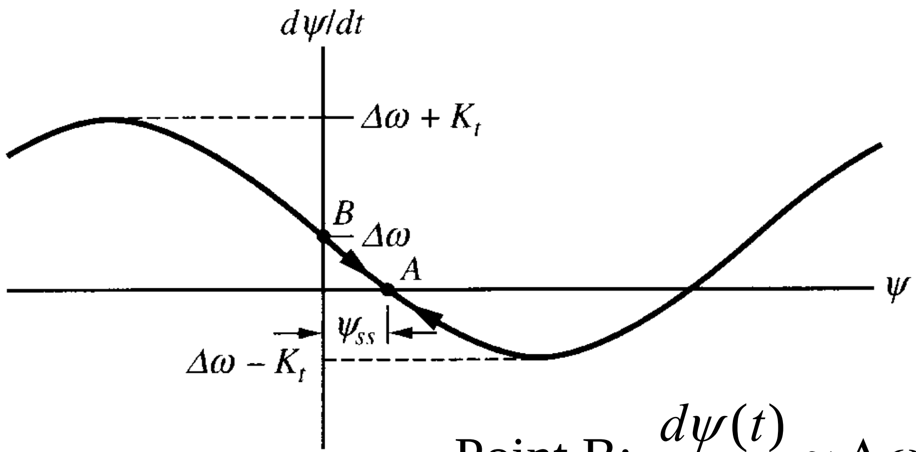


Figure 3.49
Phase-plane plot.

Point B: $\dfrac{d\psi(t)}{dt} \approx \Delta\omega$ (Step message applies)

(1) $\dfrac{d\psi(t)}{dt} > 0 \rightarrow \psi(t)$ increases $\rightarrow \sin\psi(\mathrm{t})$ increases

$\boxed{\dfrac{d\psi(t)}{dt} + K_t \sin\psi(t) = \Delta\omega} \rightarrow \dfrac{d\psi(t)}{dt}$ decreases $\rightarrow \dfrac{d\psi(t)}{dt}$ becomes negative

$\dfrac{d\psi(t)}{dt} = 0 \leftrightarrow$ point A

(2) $\dfrac{d\psi(t)}{dt} < 0 \rightarrow \psi(t)$ decreases $\rightarrow \sin\psi(\mathrm{t})$ decreases

$\boxed{\dfrac{d\psi(t)}{dt} + K_t \sin\psi(t) = \Delta\omega} \rightarrow \dfrac{d\psi(t)}{dt}$ increases $\rightarrow \dfrac{d\psi(t)}{dt}$ becomes positive

$\Rightarrow$ Point A is a locally stable point.

Remarks on Step Input

- **Steady state error**

  In this case, at Point A, $\frac{d\psi(t)}{dt} = 0$ (no frequency error)

  As $t \to \infty$, $\psi(t) = \psi_{SS} \neq 0$ (phase error exists)

- **Lock range**

  This system converges to Point A if $\Delta\omega < K_t$, K_t is the *lock range*.

  (If $\Delta\omega > K_t$, $\frac{d\psi(t)}{dt} = \Delta\omega - K_t > 0$. (when $\sin\psi(t) = 1$)

  The phase-plane plot does not intersect

  with the $\frac{d\psi(t)}{dt} = 0$ axis.)

Numerical Examples

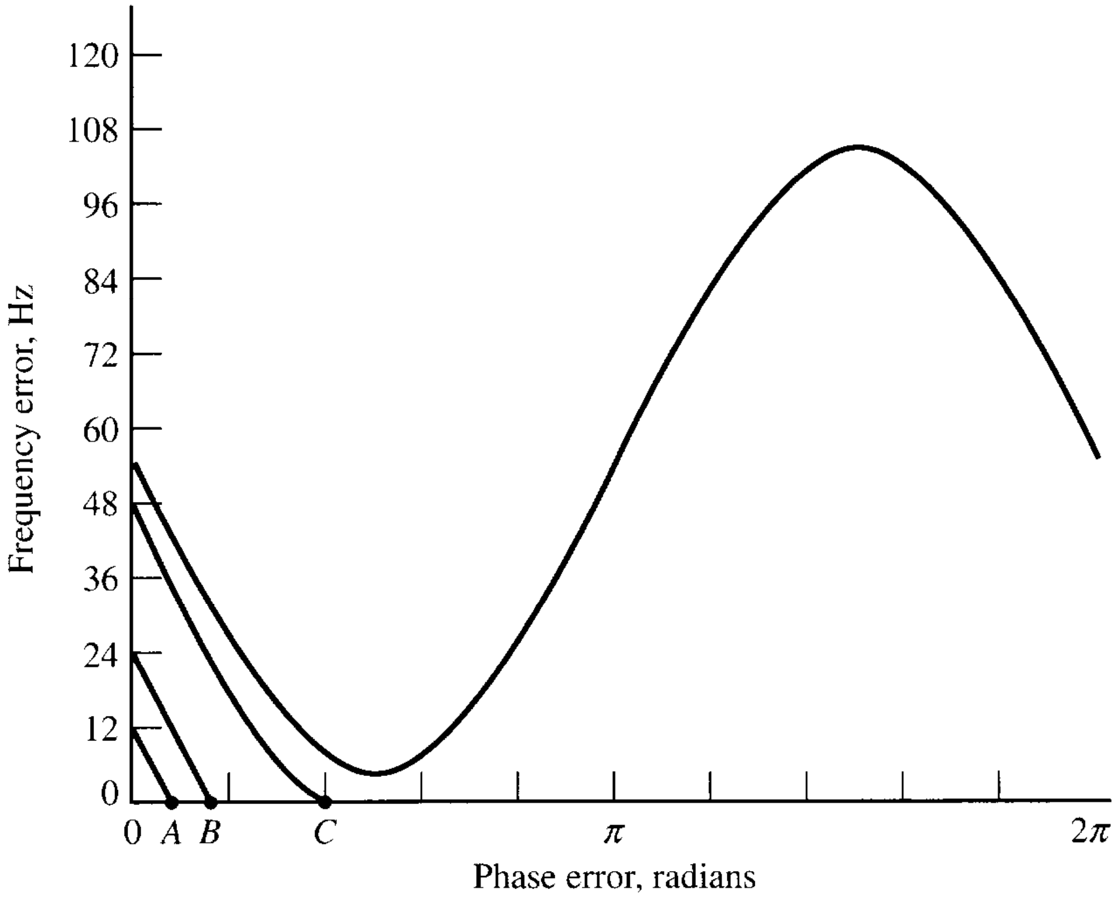


Figure 3.50
Phase-plane plot of first-order PLL for several initial frequency errors.

$K_t = 2\pi(50)$
$\Delta\omega =$
$2\pi(12)$,
$2\pi(24)$,
$2\pi(48)$,
$2\pi(55)$

Linear Model

- The detector model is a linear approximation. Now we can do freq-domain analysis

$$\sin[\phi(t)-\theta(t)] \approx \phi(t)-\theta(t)$$

$$\theta(t) = K_t \int^t (\phi(\alpha)-\theta(\alpha))d\alpha$$

$$\frac{d\theta(t)}{dt} = K_t(\phi(t)-\theta(t)) \quad \Rightarrow \quad s\cdot\Theta(s) = K_t(\Phi(s)-\Theta(s))$$

$$\Rightarrow H(s) = \frac{\Theta(s)}{\Phi(s)} = \frac{K_t}{s+K_t} \quad \Rightarrow \quad h(t) = K_t e^{-K_t t}u(t).$$

First-order PLL

Step Input

Example: A frequency jump $\dfrac{d\phi(t)}{dt} = K_f A u(t)$

$$s \cdot \Phi(s) = K_f A \frac{1}{s} \qquad \Rightarrow \qquad \Phi(s) = K_f A \frac{1}{s^2}$$

$$\Theta(s) = \frac{K_t}{s + K_t} \Phi(s) = \frac{A K_f K_t}{s^2 (s + K_t)}.$$

- How about error? $\psi(t) = \phi(t) - \theta(t)$

Error Analysis

Phase: $\Psi(s) = \Phi(s) - \Theta(s)$

$$= \frac{AK_f}{s^2}(1 - \frac{K_t}{s+K_t}) = \frac{AK_f}{s}(\frac{1}{s+K_t}) = \frac{AK_f}{K_t}(\frac{1}{s} - \frac{1}{s+K_t}).$$

$$\psi(t) = \frac{AK_f}{K_t}(u(t) - e^{-K_t t}u(t)). \text{ As } t \to \infty,\ \psi(t) = \frac{AK_f}{K_t}u(t).$$

Frequency: $s \cdot \Psi(s) = \frac{sAK_f}{s}(\frac{1}{s+K_t}) = \frac{AK_f}{s+K_t}.$

$$\frac{d\psi(t)}{dt} = AK_f e^{-K_t t}u(t). \text{ As } t \to \infty,\ \frac{d\psi(t)}{dt} = 0.$$

Note: $K_t \uparrow$, $\psi_{SS} \downarrow$ and $\frac{d\psi(t)}{dt}$ converges quicker!

It is clear that $H(s) \approx 1$ as $K_t \to \infty$

First-order PLL Summary

1. Limited lock range: $\Delta\omega < K_t$.
2. Nonzero steady state error $\psi_{SS} \propto \dfrac{1}{K_t}$
3. The complete system loop gain is

$$K_t = \frac{1}{2} A_C \cdot \mu \cdot A_v \cdot k_d \cdot K_V$$

4. K_t also controls the bandwidth of PLL

   → 3dB point = K_t

$$\boxed{H(s) = \frac{K_t}{s + K_t}}$$

5. A large K_t is impractical because (a) hardware implementation issues and (b) noise increases due to the wide bandwidth. → higher-order PLL.

2^{nd} –order PLL

<Case 2> Perfect 2^{nd}-order PLL ← linear model

Loop filter $= F(s) = (s+a)/s$ (1st order)

$$s \cdot \Theta(s) = K_t F(s)[\Phi(s) - \Theta(s)].$$

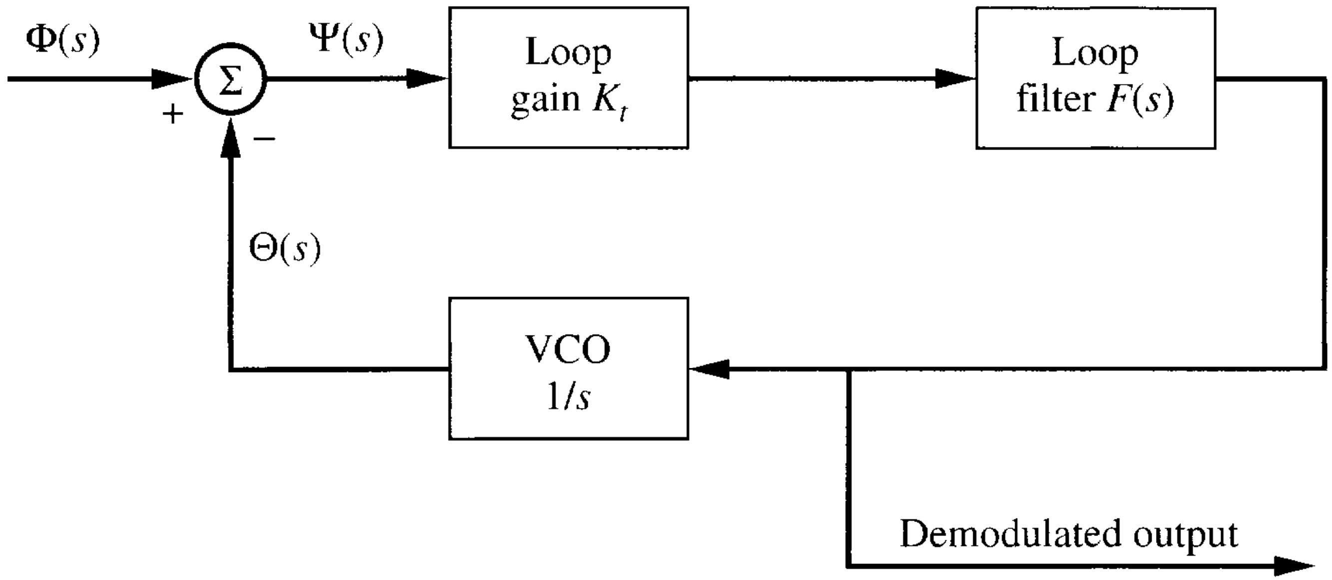


Figure 3.48
Linear PLL model in the frequency domain.

2^{nd}-order PLL Analysis

$$H(s) = \frac{\Theta(s)}{\Phi(s)} = \frac{K_t F(s)}{s + K_t F(s)} = \frac{K_t (s + a)}{s^2 + K_t s + K_t a}.$$

Then, the difference between the input and the tracking output.

$$\Psi(s) = \Phi(s) - \Theta(s) = \Phi(s) - H(s)\Phi(s) = \Phi(s)(1 - H(s))$$

$$\frac{\Psi(s)}{\Phi(s)} = 1 - H(s) = \frac{s}{s + K_t F(s)} = \frac{s^2}{s^2 + K_t s + K_t a} = \frac{s^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

where $\omega_n = \sqrt{K_t a}$ (natural frequency)

$\varsigma = \frac{1}{2}\sqrt{\frac{K_t}{a}}$ (damping factor).

This is a general 2nd-order transfer function.

Step Input

$$\frac{d\phi(t)}{dt} = \Delta\omega \cdot u(t). \ \ \text{(in time domain)}$$

$$\Phi(s) = \frac{\Delta\omega}{s^2}. \ \ \text{(in } s \text{ domain)}$$

$$\Rightarrow \Psi(s) = \frac{s^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}\frac{\Delta\omega}{s^2} = \frac{\Delta\omega}{s^2 + 2\varsigma\omega_n s + \omega_n^2}.$$

$$\Rightarrow \psi(t) = \frac{\Delta\omega}{\omega_n\sqrt{1-\varsigma^2}} e^{-\varsigma\omega_n t} \sin(\omega_n\sqrt{1-\varsigma^2} \cdot t) \ \ \text{for} \ \ \varsigma < 1.$$

$$\therefore \psi(t) \to 0 \text{ as } t \to \infty \text{ (no steady state frequency error).}$$

- *Remarks*: Pull-in range = ∞! But has *cycle-slipping.*
  Cycle-slipping: Steady state phase error is $2\pi \cdot m$ (rad).

Simulated PLL behavior

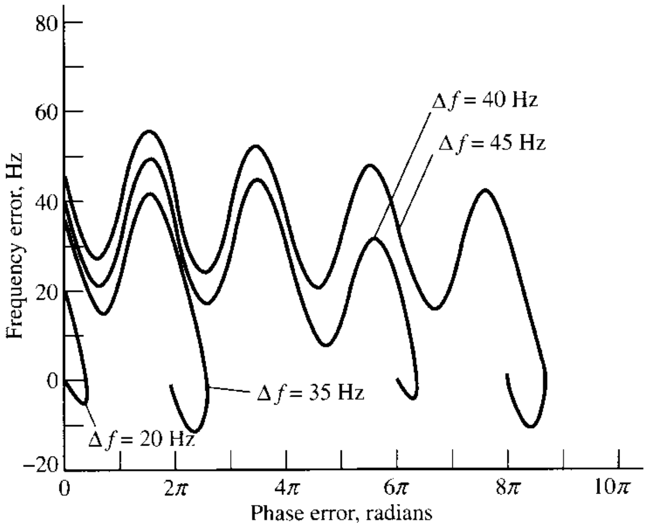


Figure 3.51
Phase-plane plot for second-order PLL.

1) Δf =20Hz
2) Δf =35Hz
3) Δf =40Hz
4) Δf =45Hz

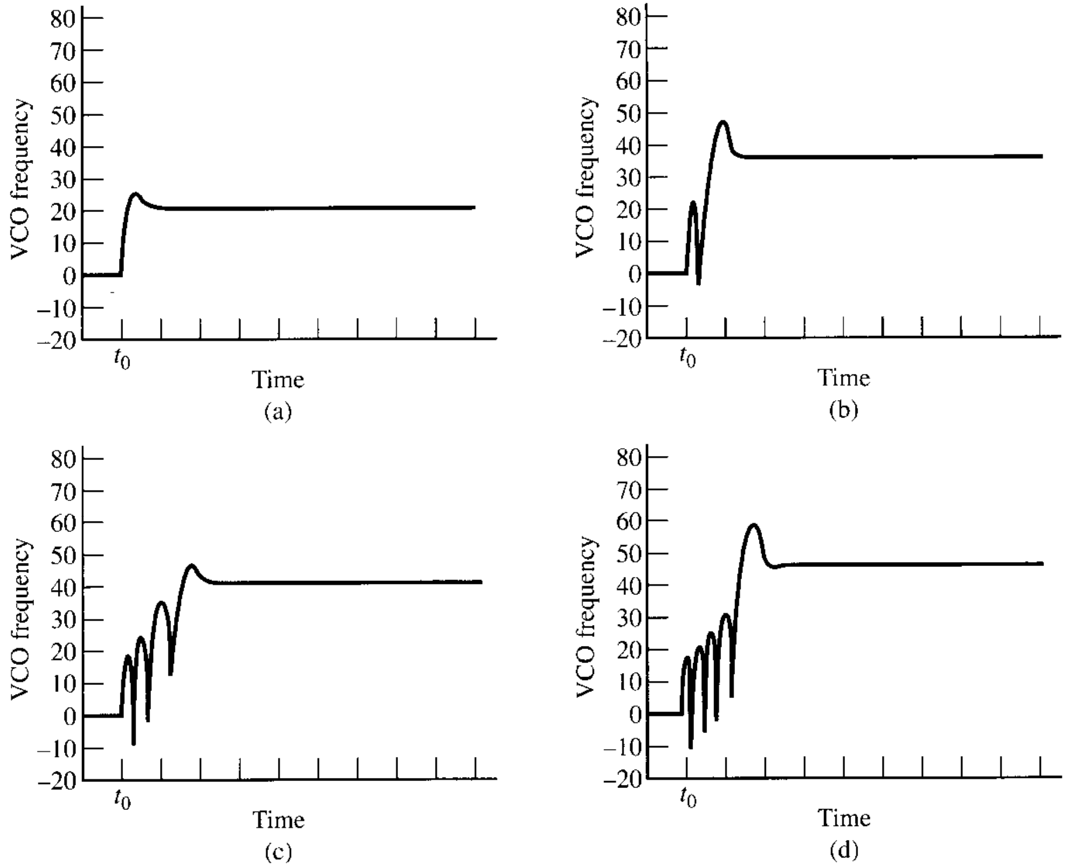


Figure 3.52
voltage-controlled oscillator frequency for four values of input frequency step. (a) VCO frequency for $\Delta f = 20$ Hz. (b) VCO frequency for $\Delta f = 35$ Hz. (c) VCO frequency for $\Delta f = 40$ Hz. (d) VCO frequency for $\Delta f = 45$ Hz.

PLL Applications

- Frequency multiplier
- Frequency divider
- FM demodulation

Frequency Multiplier

- Generate the harmonics of the input and the VCO tracks one of the harmonics.

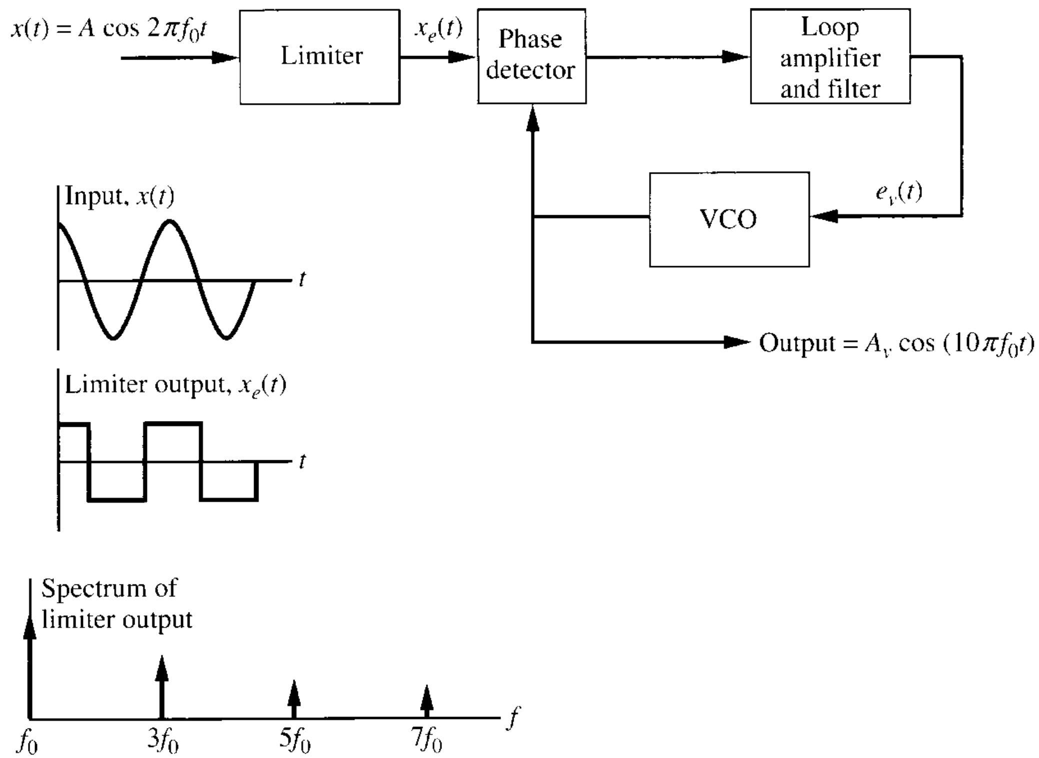


Figure 3.54
Phase-locked loop used as a frequency multiplier.

Frequency Divider

- Generate the harmonics of the VCO output. One of the harmonics tracks the input.

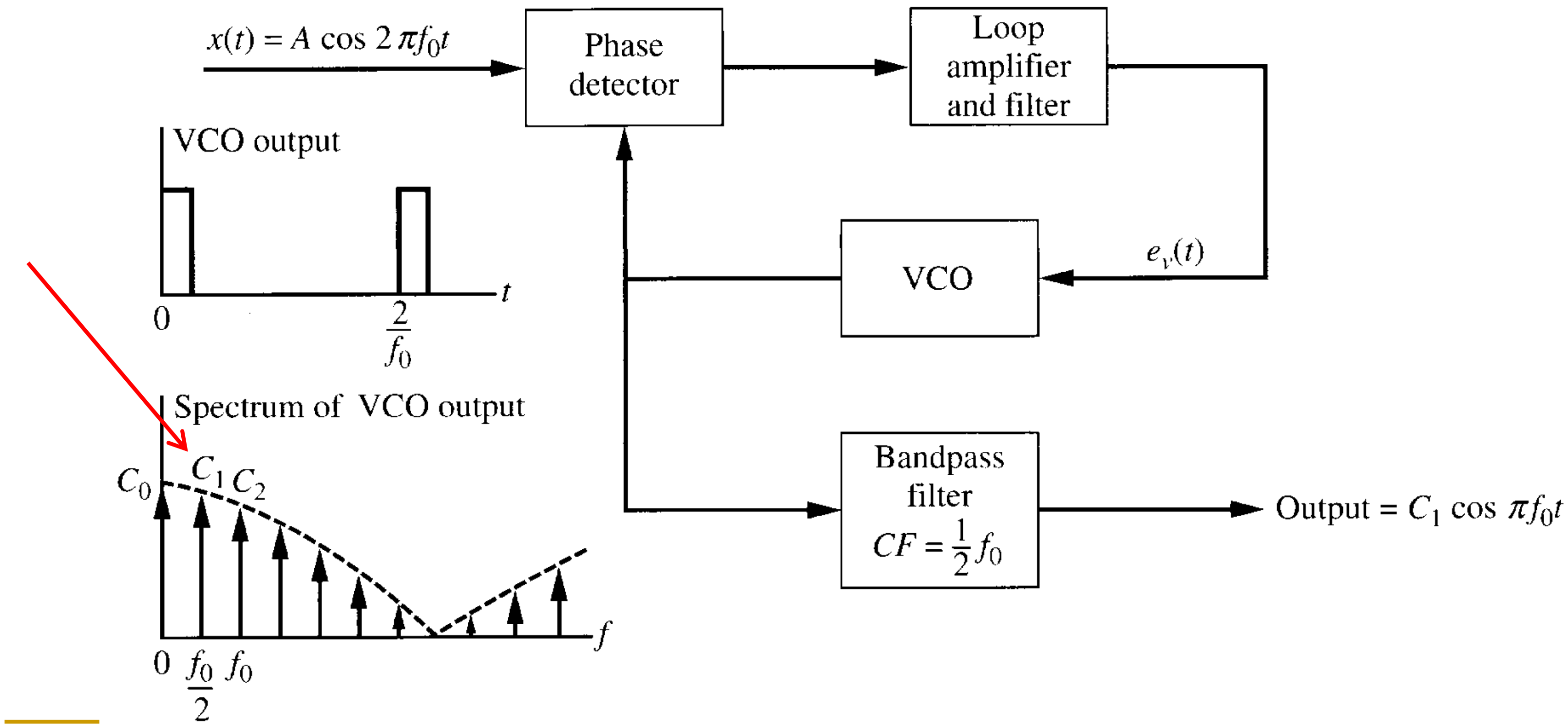
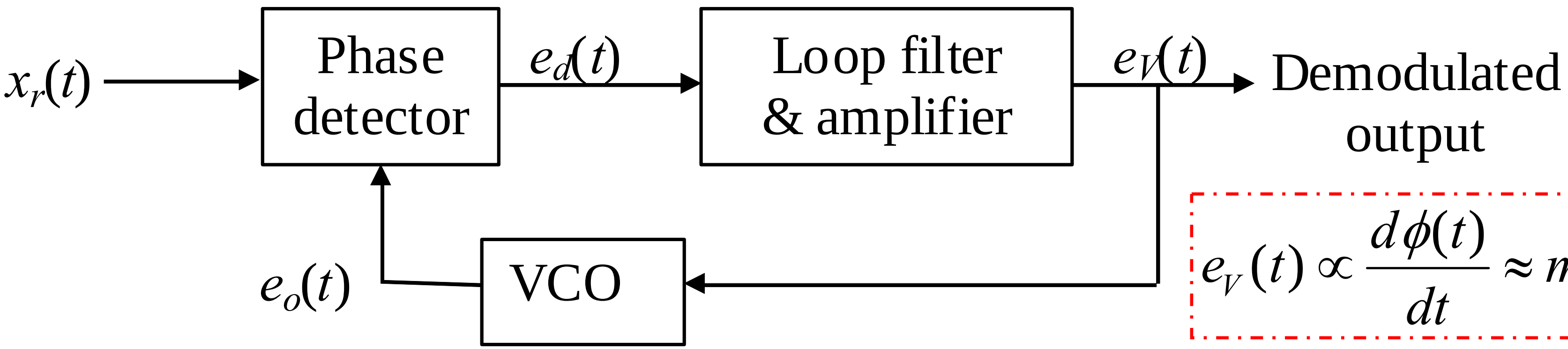


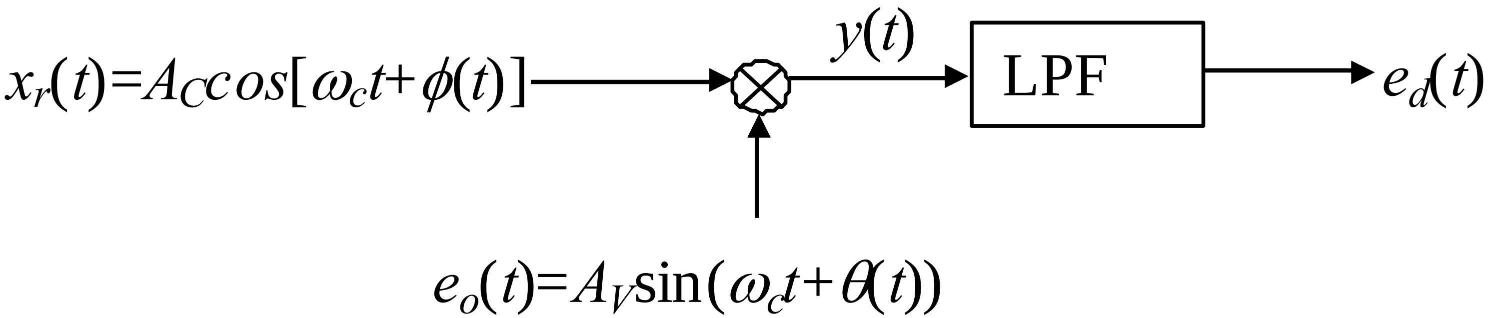
Figure 3.55
Phase-locked loop used as a frequency divider.

FM Demodulator



$$e_V(t) \propto \frac{d\phi(t)}{dt} \approx m(t)$$

- Implementation of "phase detector"



FM Demodulator (2)

$$y(t) = x_r(t) \cdot e_o(t) = \frac{A_C A_V}{2} \sin[2\omega_c t + \phi(t) + \theta(t)] -$$

$$\frac{A_C A_V}{2} \sin[\phi(t) - \theta(t)]$$

$$e_d(t) \cong -\frac{A_C A_V}{2} \sin[\phi(t) - \theta(t)]$$

When $[\phi(t) - \theta(t)]$ is small, $\sin[\phi(t) - \theta(t)] \approx \phi(t) - \theta(t)$.

Costas Phase-Lock Loops

- Recover the carrier in DSB and digital communication systems

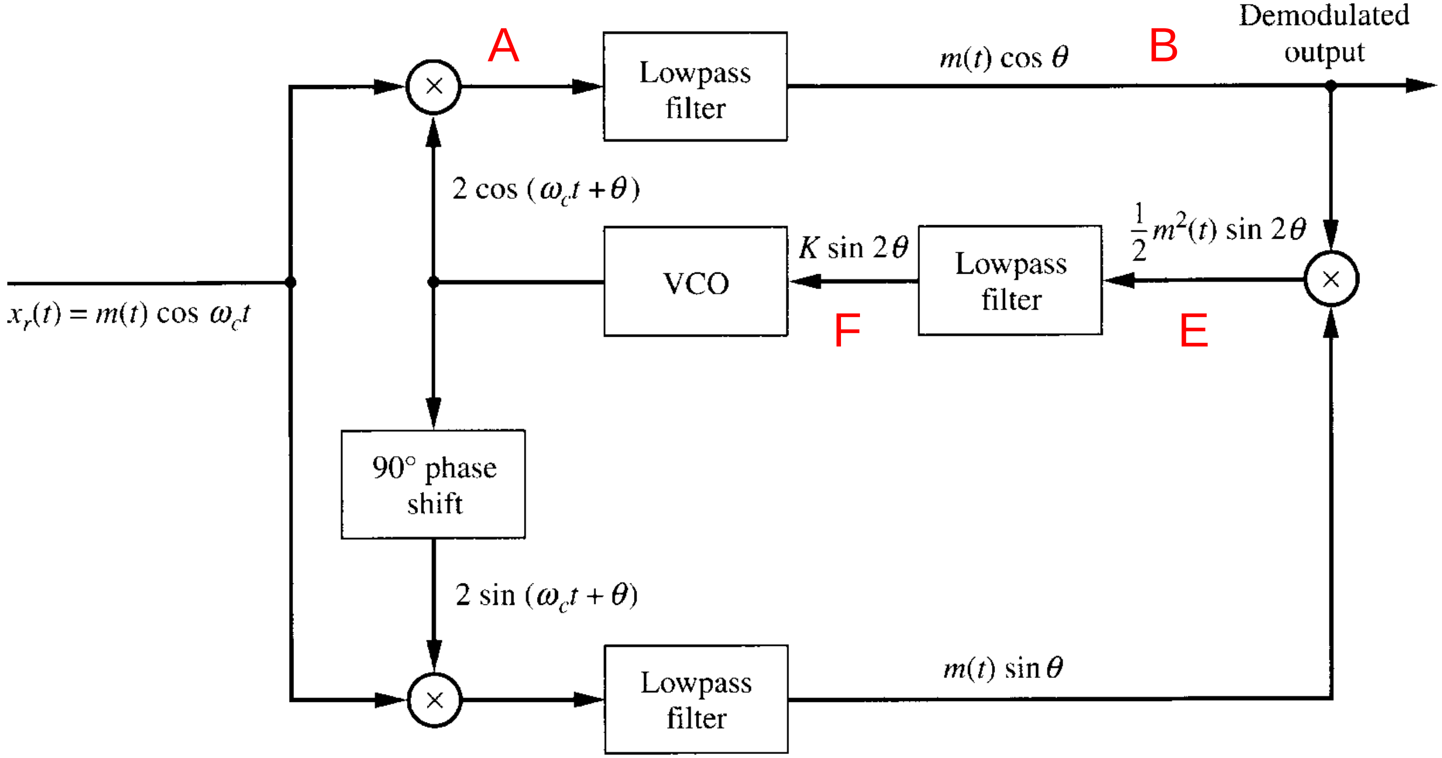


Figure 3.53
Costas PLL.

Costas PLL (2)

- Ideally, $\theta \approx 0$, I-channel output (B) $\approx m(t)$. (Q-channel output (D) ≈ 0)

- If $\theta(t) \neq 0$,

Nearly constant low freq comp

$$\text{signal at E} = m(t)\cos\theta \cdot m(t)\sin\theta = m^2(t)\cdot\frac{1}{2}\sin 2\theta$$

$$\text{signal at F} \approx K\cdot\sin 2\theta \approx 2K\theta, \text{ when } \theta \text{ is small}$$

- VCO output is adjusted so that $\theta \to 0$.

Parameters

- **Lock range**: This is the freq range within which a PLL locks within one single beat note. Normally, the operation freq range of a PLL is restricted to the lock range.
- **Pull-in range**: within which a PLL will always become locked, but the process can be rather slow.
- **Hold range:** in which a PLL can *statically* maintain phase tracking. A PLL is conditionally stable only within this range.
- **Pull-out range**: The *dynamic* limit for stable operation of a PLL. If tracking is lost within this rage, a PLL normally will lock again.

Parameters

- **Lock time**: The time PLL needs to get locked when the acquisition process is a (fast) lock-in process
- **Pull-in time**: The time PLL needs to get locked when the acquisition process is a (slow) pull-in process

Hold range ≧ Pull-in ≧ Pull-out ≧ Lock range (R. Best, Phase-locked Loops; Fig 2.29)