Principles of Communications Lecture 5: Analog Modulation Techniques (3)

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Outlines

- Linear Modulation
- Angle Modulation
- Interference
- Feedback Demodulators
- Analog Pulse Modulation
- Delta Modulation and PCM
- Multiplexing

Angle Modulation

- Modulated signals have constant amplitude; information is embedded in the **phase**.

General form: $x_c(t) = A_C \cos[\omega_c t + \phi(t)]$

$\bullet$ Phase deviation: $\phi(t)$

$\bullet$ Frequency deviation: $\dfrac{d\phi(t)}{dt}$

PM: $\phi(t) = K_P \cdot m(t)$ ($m(t)$ is the message.)

FM: $\dfrac{d\phi(t)}{dt} = K_F \cdot m(t)$

$$\phi(t) = K_F \int_{t_0}^{t} m(\alpha)d\alpha + \phi_0 = 2\pi f_d \int_{t_0}^{t} m(\alpha)d\alpha + \phi_0$$

Example of PM and FM Waveforms

$$\text{PM: } x_c(t) = A_C \cos[\omega_c t + K_P m(t)]$$

$$\text{FM: } x_c(t) = A_C \cos[\omega_c t + 2\pi f_d \int_{t_0}^{t} m(\alpha) d\alpha]$$

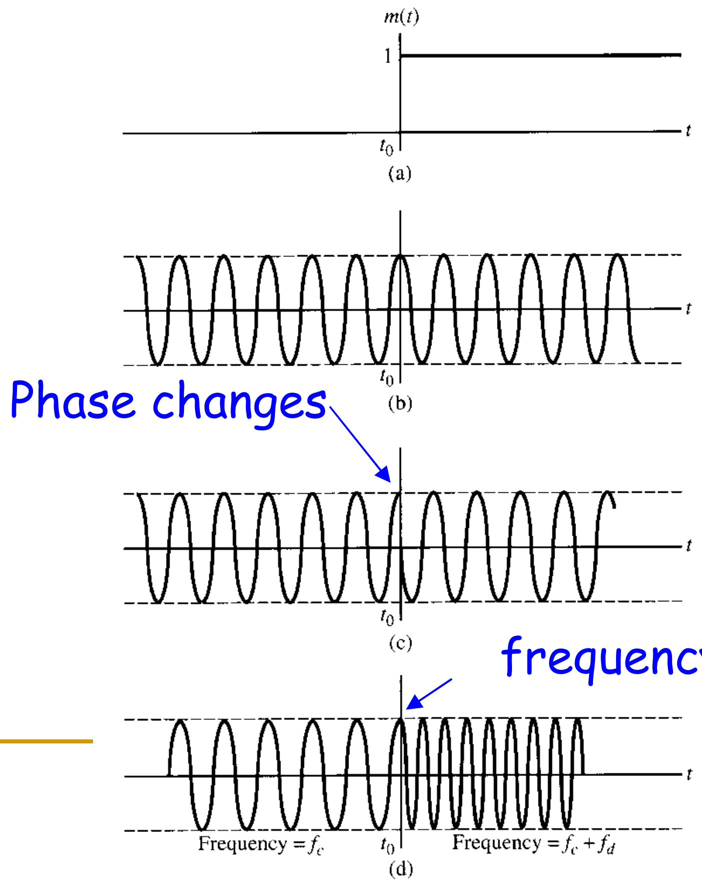


Figure 3.20
Comparison of PM and FM modulator outputs for a unit-step input. (a) Message signal. (b) Unmodulated carrier. (c) Phase modulator output $\left(k_p = \frac{1}{2}\pi\right)$. (d) Frequency modulator output.

Another Example

- Sinusoidal message

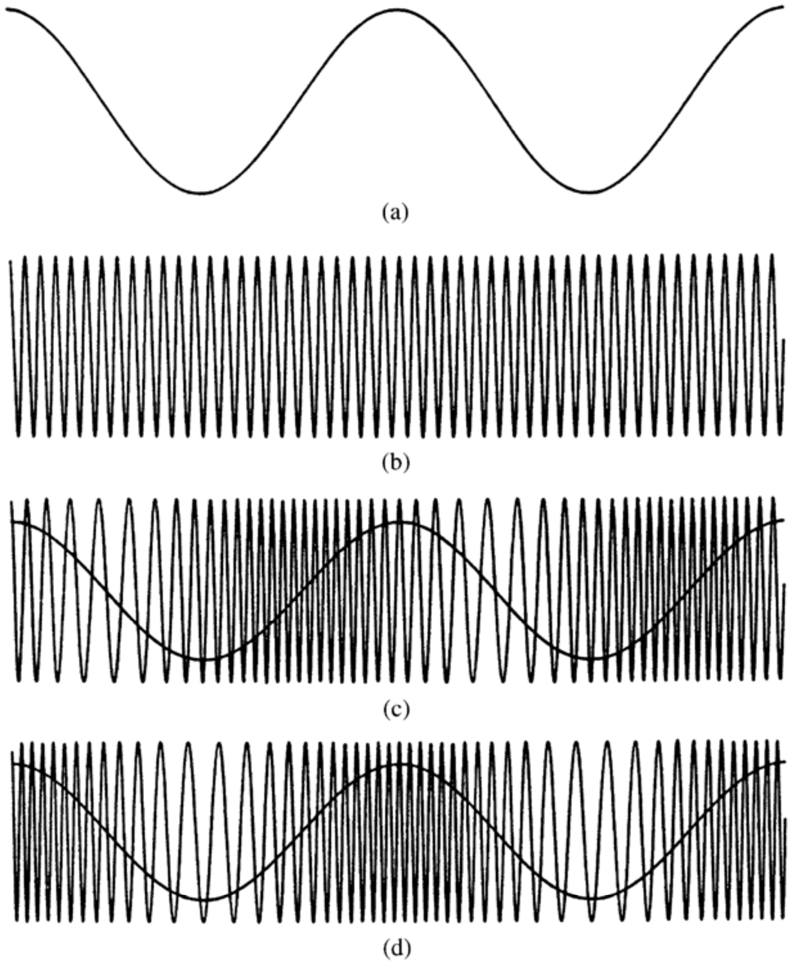


Figure 3.21
Angle modulation with sinusoidal message signal. (a) Message signal $m(t)$. (b) Unmodulated carrier $A_c \cos(2\pi f_c t)$. (c) Output of phase modulator with $m(t)$. (d) Output of frequency modulator with $m(t)$.

Angle Modulation Analysis

- Narrow-band modulated signals

  Wide-band modulated signals

  -- Message: single tone; general signal

- Narrowband Angle Modulation

$$x_c(t) = A_C \cos[\omega_c t + \varphi(t)] = \mathrm{Re}\{A_C e^{j\omega_c t} e^{j\varphi(t)}\}$$

$$= \mathrm{Re}\{A_C e^{j\omega_c t}(1 + j\varphi(t) + \frac{(j\varphi(t))^2}{2!} + \cdots)\}$$

$$\approx \mathrm{Re}\{A_C e^{j\omega_c t}(1 + j\varphi(t))\}$$

Tayler series expansion..

$$\Rightarrow x_c(t) \approx A_C \cos\omega_c t - A_C \varphi(t) \sin\omega_c t$$

Narrow-band Signal Generation

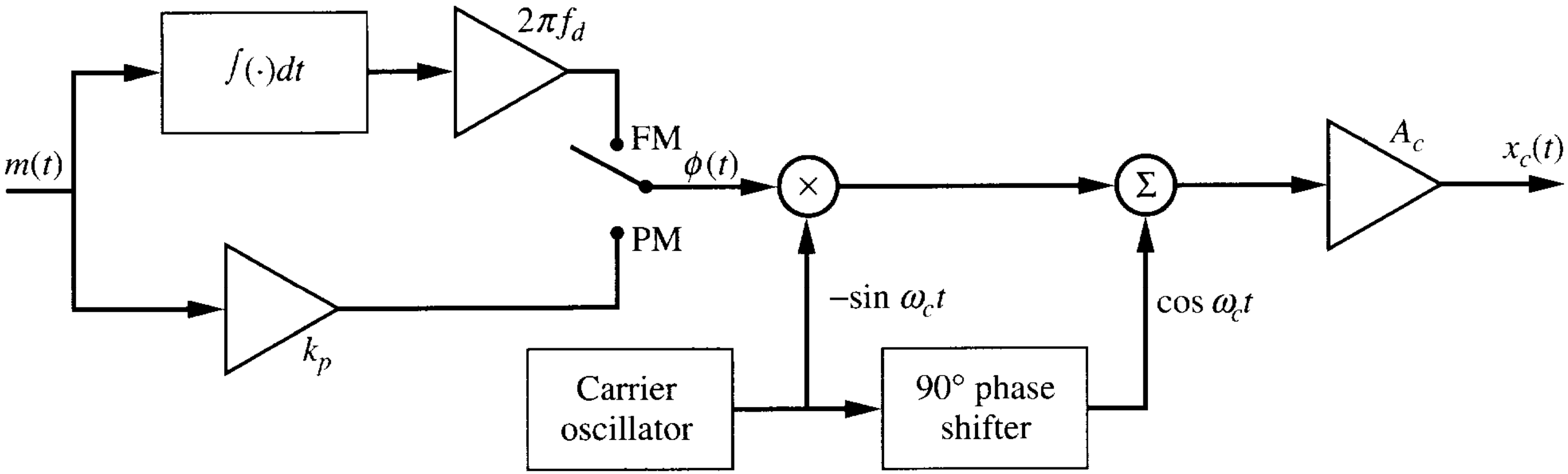
- Narrowband angle-mod signals ~ AM signal except that $m(t)$ multiplies a 90º phase-shifted carrier
- BW of $\varphi(t)$ = W → BW of $x_c(t)$ = 2W (*narrow!*)
- 

Figure 3.22
Generation of narrowband angle modulation.

Narrow-band Signal Example

- $m(t)=\cos(2\pi f_m t)$ AM vs. PM

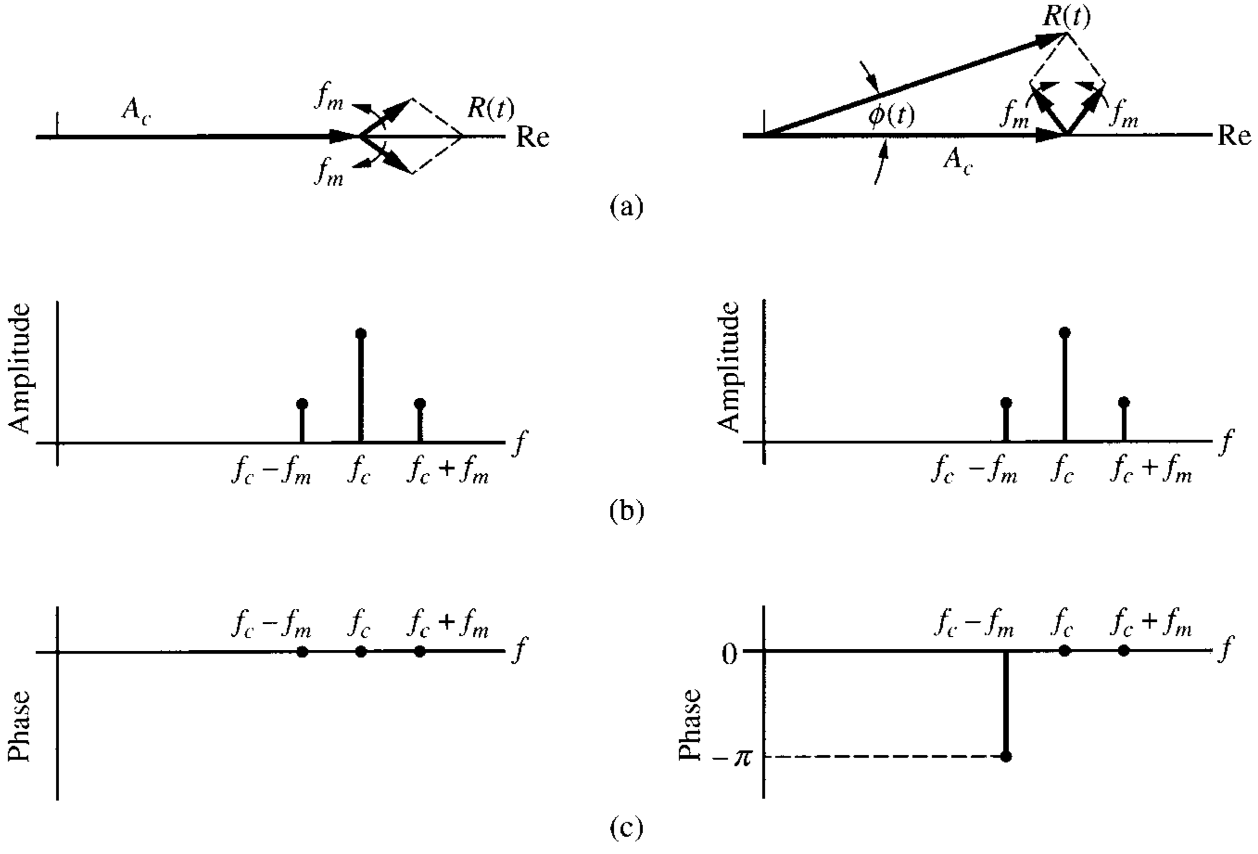


Figure 3.23
Comparison of AM and narrowband angle modulation. (a) Phasor diagrams. (b) Single-sided amplitude spectra. (c) Single-sided phase spectra.

Wide-band Angle Modulation

- The modulated signal has a much wider BW than the original message.

  (A) Message is a *single tone* -- $m(t)=\sin(2\pi f_m t)$

- **Modulation index** β is sufficiently large

  $$\text{Let } \varphi(t) = \beta \sin \omega_m t$$

  ➔ β controls the maximum phase deviation

- *Q*: Why do we study the sinusoidal message?

Wide-band FM (Single Tone)

Let $\phi(t) = \beta \sin \omega_m t.$

$$x_c(t) = A_C \cos(\omega_c t + \beta \sin \omega_m t) = A_C \operatorname{Re}\{e^{j\omega_c t} e^{j\beta \sin \omega_m t}\}$$

FS of $e^{j\beta \sin \omega_m t} \rightarrow \; C_n = \dfrac{1}{T_0}\displaystyle\int_0^{T_0} e^{j\beta \sin \omega_m t} e^{-jn\omega_m t} dt = \dfrac{\omega_m}{2\pi}\int_{-\pi/\omega_m}^{\pi/\omega_m} e^{j\beta \sin \omega_m t} e^{-jn\omega_m t} dt$

$= \dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} e^{-j(nx-\beta \sin x)} dx \equiv J_n(\beta)$ **Bessel function** of the 1st kind (order n)

$$e^{j\beta \sin \omega_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot e^{jn\omega_m t}$$

$$\Rightarrow x_c(t) = A_C \operatorname{Re}\{e^{j\omega_c t} \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn\omega_m t}\} = A_C \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[(\omega_c + n\omega_m)t]$$

$$\Rightarrow X_c(f) = \sum_{n=-\infty}^{\infty} J_n(\beta)\delta(\omega - n\omega_m - \omega_c) + \sum_{n=-\infty}^{\infty} J_n(\beta)\delta(\omega + n\omega_m + \omega_c).$$

Bessell Function

Table 3.2 Bessel Functions

n	$\beta=0.05$	$\beta=0.1$	$\beta=0.2$	$\beta=0.3$	$\beta=0.5$	$\beta=0.7$	$\beta=1.0$	$\beta=2.0$	$\beta=3.0$	$\beta=5.0$	$\beta=7.0$	$\beta=8.0$	$\beta=10.0$
0	0.999	0.998	0.990	0.978	0.938	0.881	0.765	0.224	−0.260	−0.178	0.300	0.172	−0.246
1	0.025	0.050	0.100	0.148	0.242	0.329	0.440	0.577	0.339	−0.328	−0.005	0.235	0.043
2		0.001	0.005	0.011	0.031	0.059	0.115	0.353	0.486	0.047	−0.301	−0.113	0.255
3				0.001	0.003	0.007	0.020	0.129	0.309	0.365	−0.168	−0.291	0.058
4						0.001	0.002	0.034	0.132	0.391	0.158	−0.105	−0.220
5								0.007	0.043	0.261	0.348	0.186	−0.234
6								0.001	0.011	0.131	0.339	0.338	−0.014
7									0.003	0.053	0.234	0.321	0.217
8										0.018	0.128	0.223	0.318
9										0.006	0.059	0.126	0.292
10										0.001	0.024	0.061	0.207
11											0.008	0.026	0.123
12											0.003	0.010	0.063
13											0.001	0.003	0.029
14												0.001	0.012
15													0.005
16													0.002
17													0.001

FM Spectrum (Tone)

- $x_c(t)$ has a line-spectrum (FS)

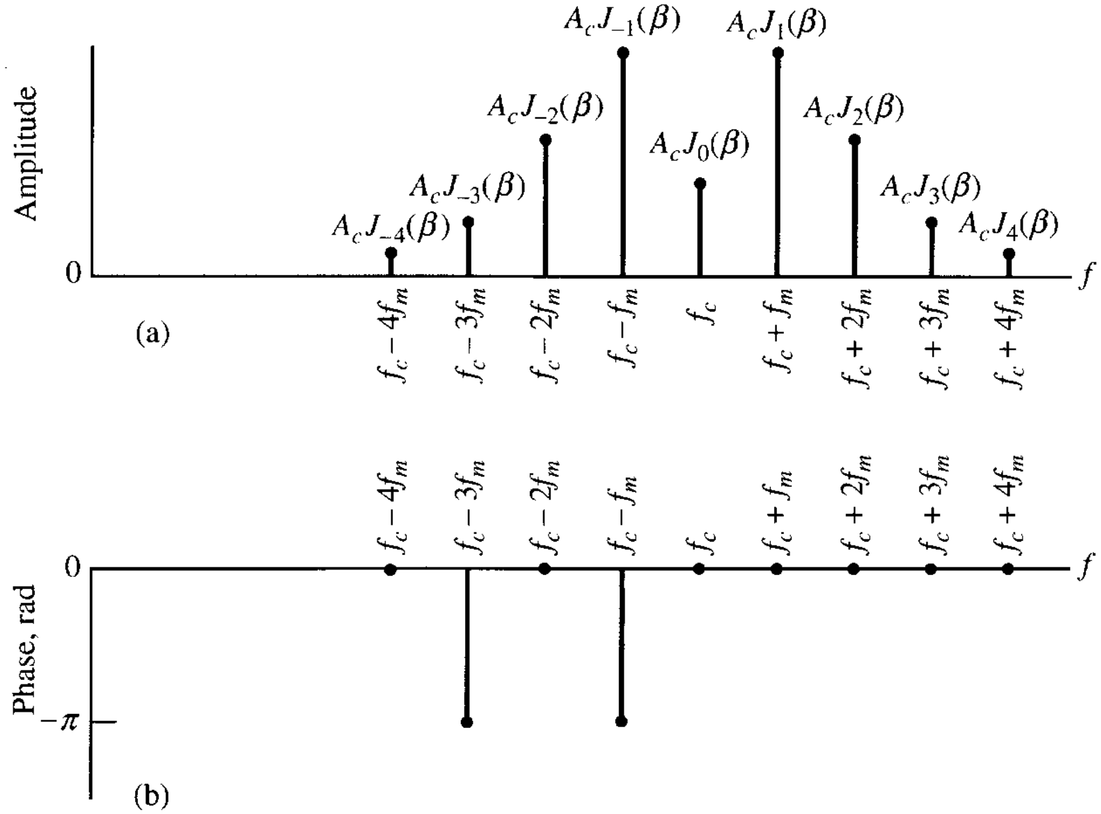


Figure 3.24
Spectra of an angle-modulated signal. (a) Single-sided amplitude spectrum. (b) Single-sided phase spectrum.

Properties of $J_n(\beta)$

1. $J_n(\beta)$ are real-valued
2. $J_{-n}(\beta) = J_n(\beta)$, n even

   $J_{-n}(\beta) = -J_n(\beta)$, n odd
3. Recursive relation: Given $J_0(\beta)$ and $J_1(\beta)$, we can derive all the $J_n(\beta)$ for n>=2. $\boxed{J_{n+1}(\beta) = \frac{2n}{\beta} J_n(\beta) + J_{n-1}(\beta)}$
4. When β is small (narrowband FM/PM)

   $J_0(\beta) \sim 1$; $J_1(\beta) \sim \beta/2$; $J_n(\beta) \sim 0$, for n>=2
5. $\boxed{\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1}$ (*Pf*) FS representation + Parseval's Thm

$J_n(\beta)$ as a Function of β

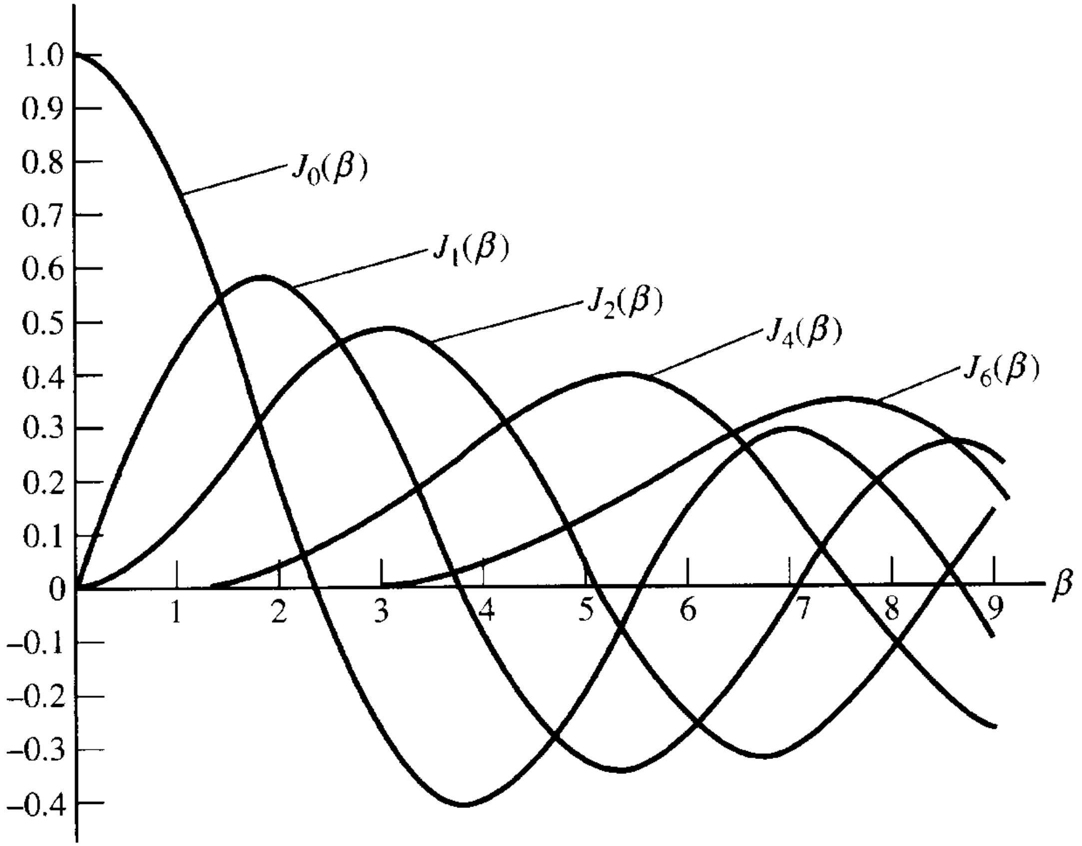


Figure 3.25
$J_n(\beta)$ as a function of β.

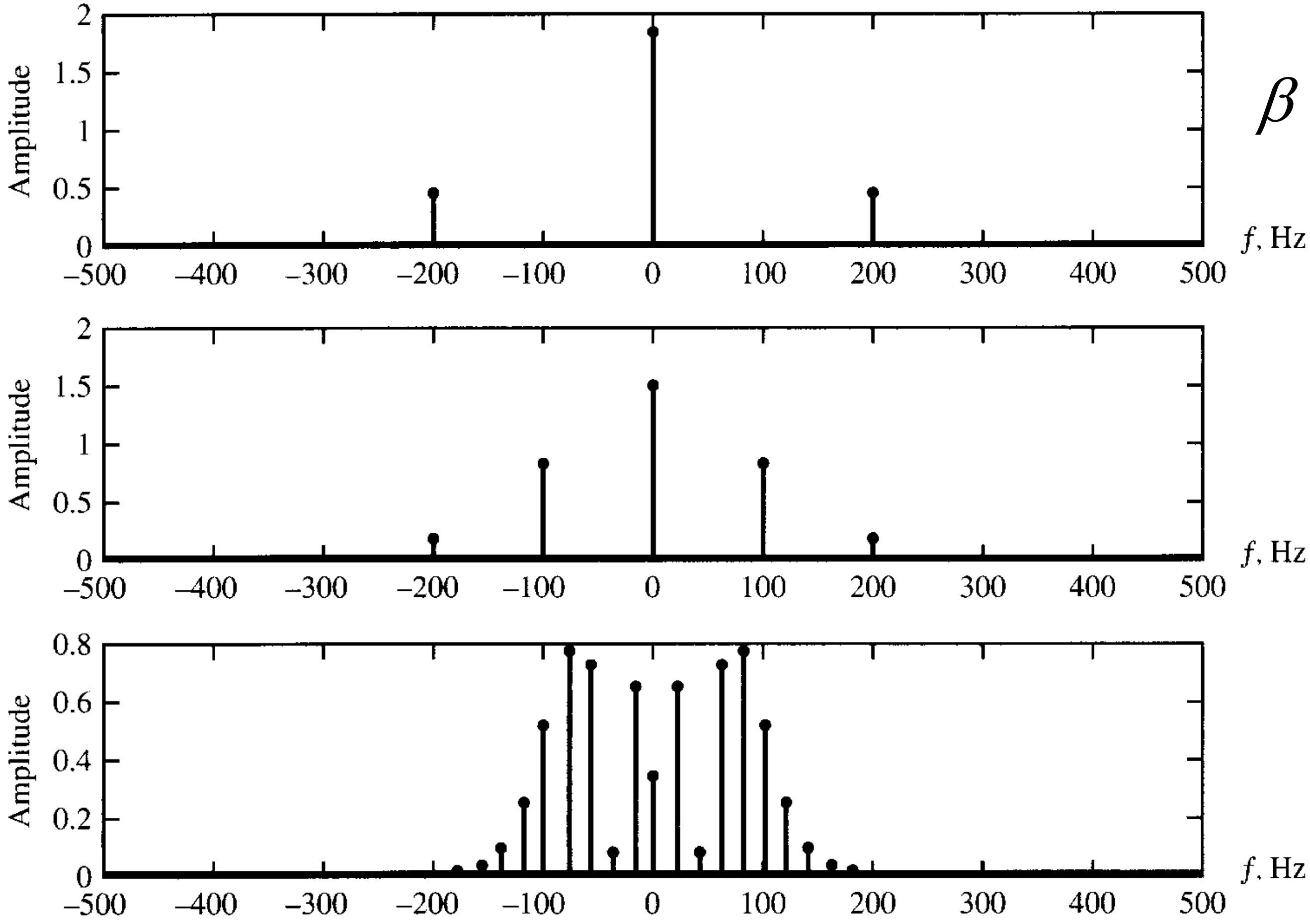
- ***Carrier nulls***: The β values such that $J_n(\beta)$=0.

Summary

- The above analysis is based on the *sinusoidal message* assumption. $\boxed{\varphi(t) = \beta \sin \omega_m t}$

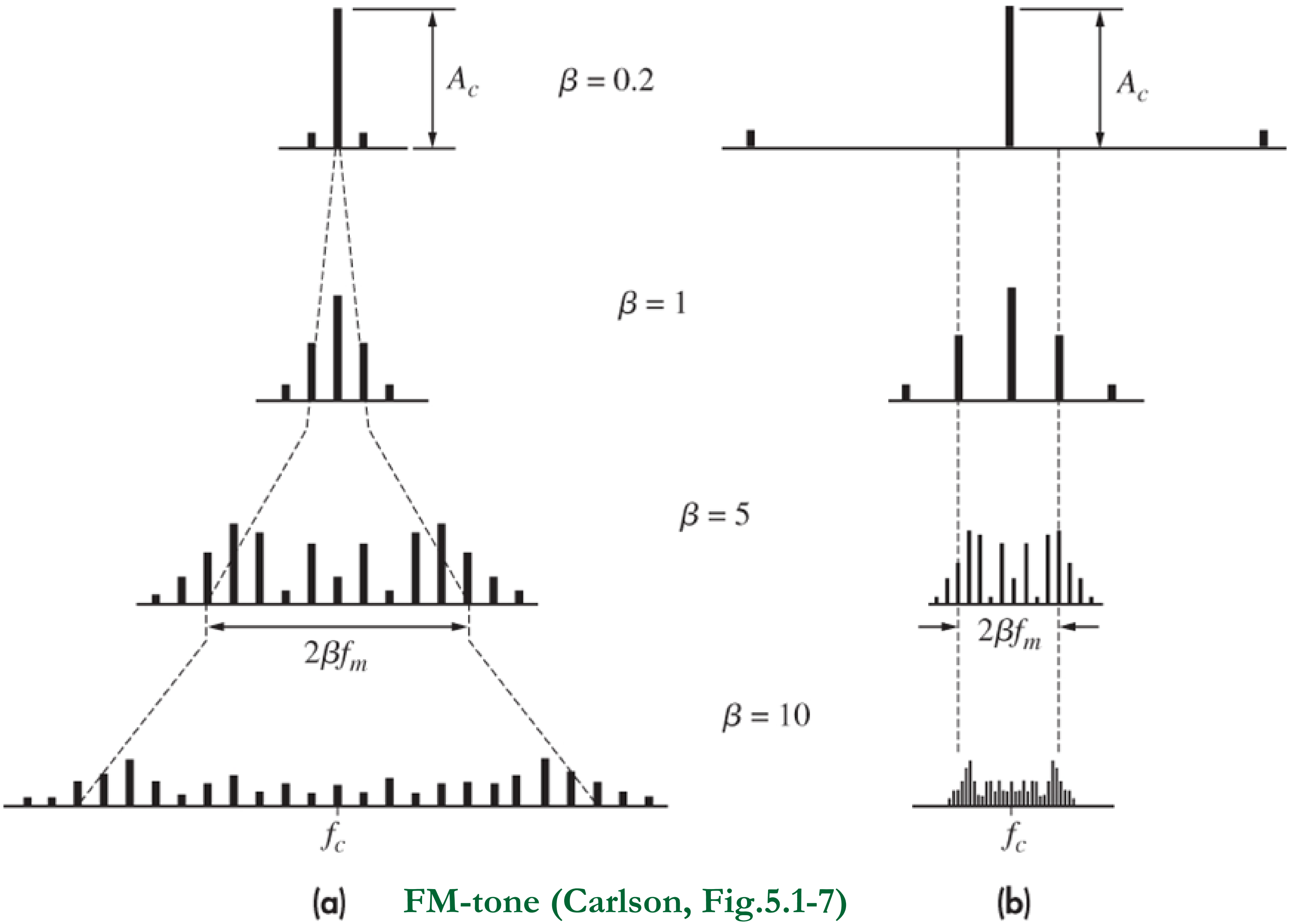
- PM: This means $m(t)=A\sin(2\pi f_m t)$
  and modulation index: β, and $K_p \equiv \beta / A$.
- FM: This means $m(t)=A\cos(2\pi f_m t)$
  and modulation index: $f_d \equiv \beta \frac{\omega_m}{2\pi A} = \beta \frac{f_m}{A}$
  Note:
  $$\varphi(t) = 2\pi f_d \int_t m(\alpha) d\alpha = 2\pi f_d \frac{A}{\omega_m} \sin \omega_m t$$

FM Spectra Examples



$$\beta f_m \approx \text{constant}$$

Figure 3.26
Amplitude spectrum of an FM complex envelope signal for increasing β and decreasing f_m.



FM-tone (Carlson, Fig.5.1-7)

Power of Angle-Mod Signals

$$\left\langle x_c^2(t) \right\rangle = A_C^2 \left\langle [\sum_n J_n(\beta)\cos(\omega_c + n\omega_m)t]^2 \right\rangle \quad \text{[cross terms are zero]}$$

$$= \frac{1}{2} A_C^2 \sum_n J_n{}^2(\beta) = \frac{1}{2} A_C^2.$$

Or,

$$\left\langle x_c^2(t) \right\rangle = \left\langle (A_C \cos(\omega_c t + \beta \sin \omega_m t))^2 \right\rangle$$

$$= \frac{1}{2} A_C^2 + \frac{A_C^2}{2} \left\langle \cos 2(\omega_c t + \beta \sin \omega_m t) \right\rangle = \frac{1}{2} A_C^2.$$

- Guess the answer? (*Hint*: Waveform)
- In general, if $m(t)$ has little dc power, this is true.

BW of Angle-Mod Signals

- Small β, $J_n(\beta) \sim 0$, for $n>=2$ → BW~2W.
- Large β, find the significant coeffs that contain most of the signal power.
- Define: "**power ratio**" $\Pr \triangleq \dfrac{k \text{ component power}}{\text{total power}} = \dfrac{\frac{1}{2}A_c^2 \sum_{n=-k}^{k} J_n^2(\beta)}{\frac{1}{2}A_c^2}$

$$= J_0^2(\beta) + 2\sum_{n=1}^{k} J_n^2(\beta)$$

- *Ex*: 98% power BW ~ $2(\beta+1)f_m$ for single tone
- Define: "**deviation ratio**"

$$D \triangleq \frac{\text{Peak frequency deviation}}{\text{BW of } m(t)} = \frac{\frac{1}{2\pi}\max\limits_t \left|\frac{d\phi(t)}{dt}\right|}{W} = \frac{f_d}{W}(\max|m(t)|),$$

$$\text{then BW} \approx 2(D+1)W. \quad \text{(Carson's rule)}$$

- $J_n(\beta) \approx \dfrac{\beta^n}{2^n n!}$ for large n Thus, $\lim_{n\to\infty} J_n(\beta) = 0$

For PM: $\phi(t) = K_P m(t)$

Special case: $m(t) = A\sin w_m t$ → $\dfrac{d\varphi(t)}{dt} = K_P A w_m \cos w_m t$

$$\max_t \left|\frac{d\varphi(t)}{dt}\right| = K_P A w_m$$

$\Rightarrow$ Let $\beta = K_P A$, $D = \dfrac{\beta f_m}{W} = \beta$

For FM: $\dfrac{d\varphi(t)}{dt} = K_f m(t) = 2\pi f_d m(t)$ $\quad D = \dfrac{f_d}{W}(\max_t |m(t)|)$

Special case: $m(t) = A\cos w_m t$ → $\max_t |m(t)| = A$

$\Rightarrow$ Let $\beta = \dfrac{Af_d}{f_m}$, $\quad D = \dfrac{Af_d}{f_m} = \beta$

(i) D << 1, BW ≈ 2W Narrowband angle-mod signal.

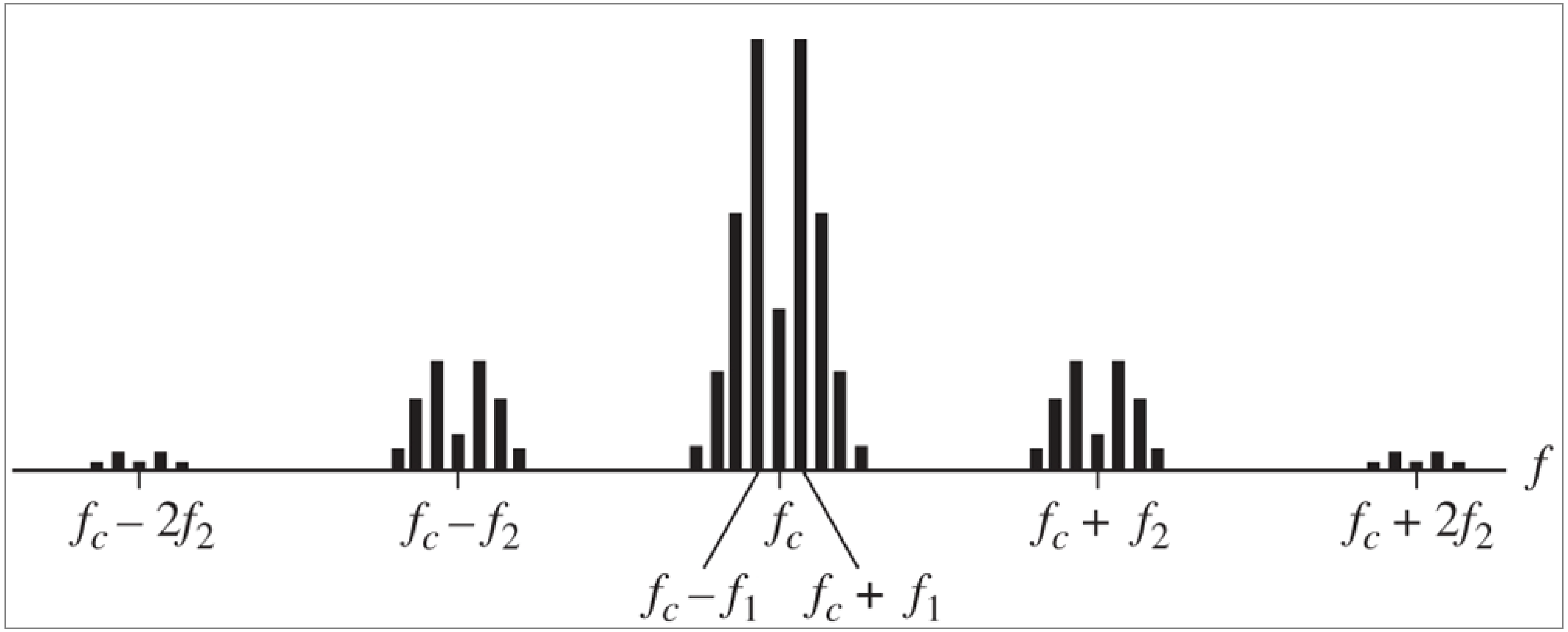
(ii) D >> 1, BW ≈ 2DW = 2f_d(max|m(t)|) Wideband angle-mod signal

Remark: **Typical value of β for FM ≈ 5.**

FM with Multi-Tone Message

(B) Two-tone case: $m(t)=A\cos(2\pi f_1 t)+B\cos(2\pi f_2 t)$

- $x_c(t)$ contains freq components at the sum and difference freqs of the modulating tones and *their harmonics*. (Carlson, Fig.5.1-9) (Z&T, p.151)



FM Signal Generation

- **Indirect FM**: narrowband-to-wideband conversion
- Two stages: 1) narrowband FM

  2) frequency multiplier

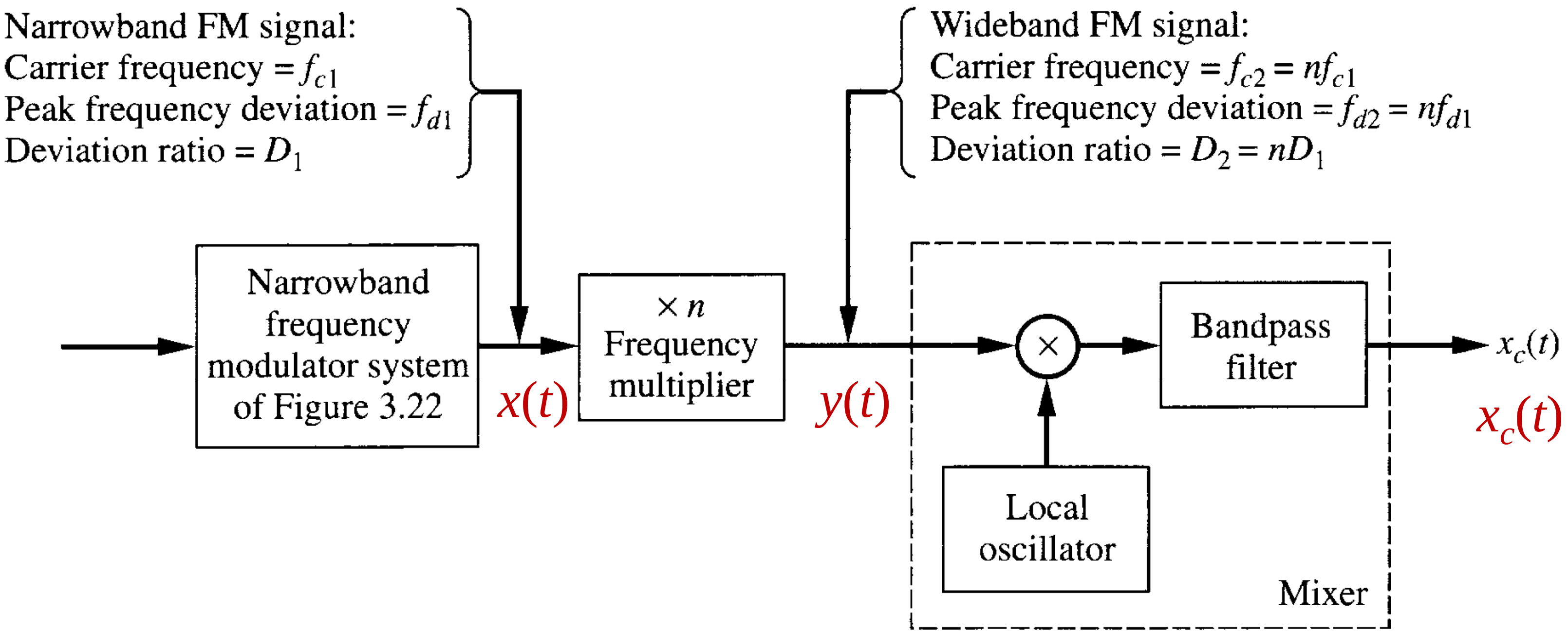


Figure 3.31
Frequency modulation utilizing narrowband-to-wideband conversion.

Indirect FM

1) Narrowband FM output: f_{c1}

$$x(t) = A_c \cos(2\pi f_0 t + \phi(t))$$

2) Freq multiplier output: (a nonilinear device) $f_{c2} = nf_{c1}$

$$y(t) = A_c \cos(2\pi n f_0 t + n\phi(t))$$

3) Mixer: frequency shift to f_c

Let the LO (local oscillator) be $e_{LO}(t)$=2cos(2 $\pi f_{LO} t$)

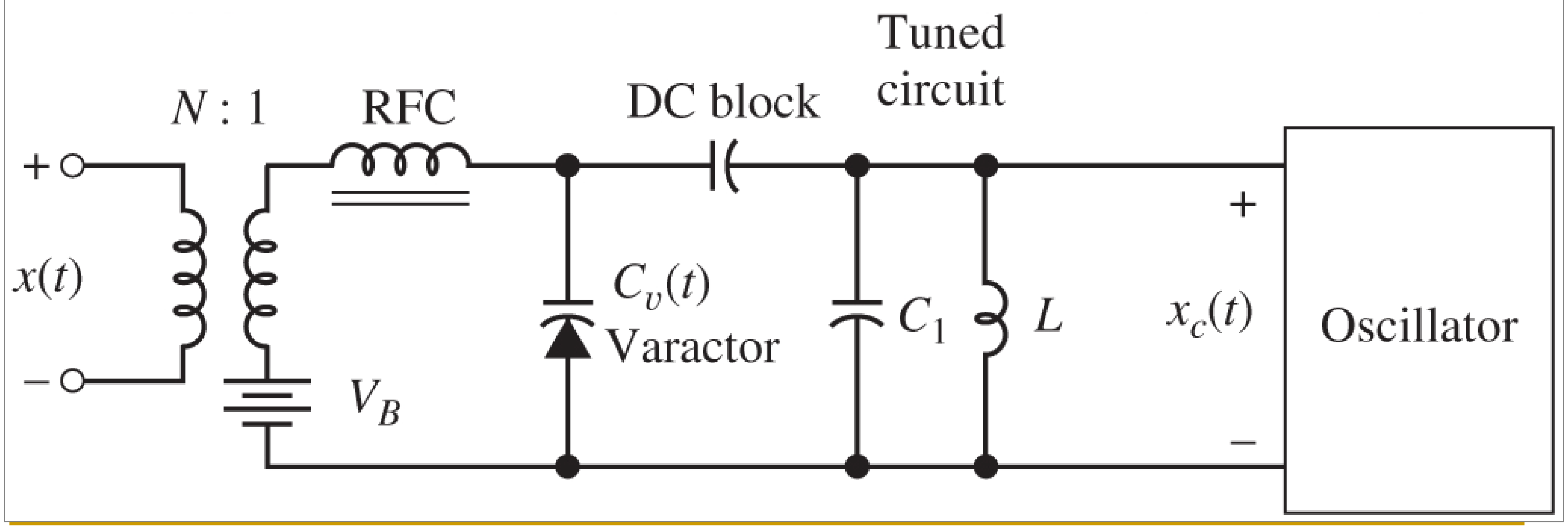
Either $f_c = nf + f_{LO}$ or $f_c = nf - f_{LO}$

$$e(t) = A_c \cos[2\pi(nf_0 + f_{LO})t + n\phi(t)]$$
$$+ A_c \cos[2\pi(nf_0 - f_{LO})t + n\phi(t)]$$
$$\rightarrow \textbf{BPF} \rightarrow x_c(t) = A_c \cos[2\pi f_c t + n\phi(t)]$$

Direct FM

- Use **voltage-controlled oscillator** (VCO) such as *variable-reactance* device (Carlson, p.233~)
- A signal-controlled capacitance device (varactor diode)

$$C(t) = C_0 + Cx(t)$$



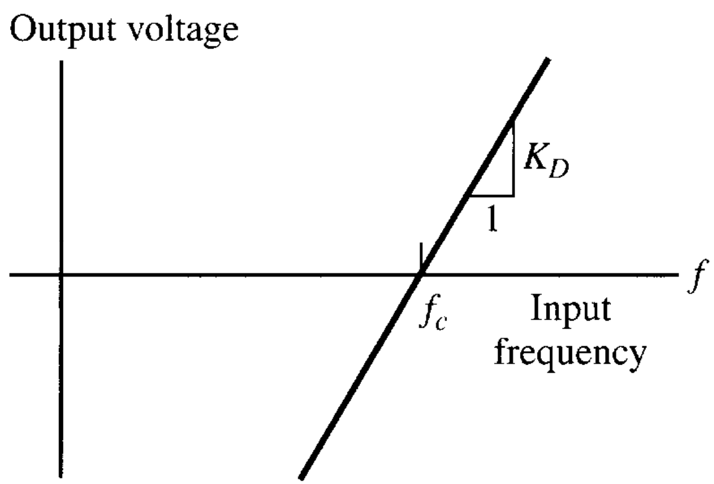
FM Signal Demodulation

- Ideal **frequency discriminator**: a device that yields an output proportional to the frequency deviation of the input.

$$\text{Received signal: } x_r(t) = A_c \cos(\omega_c t + \phi(t)).$$

$$\text{Descriminator: } y_D(t) = \frac{1}{2\pi} K_D \frac{d\phi(t)}{dt}.$$

Figure 3.32
Ideal discriminator characteristic.



FM Demodulator **(Approximation)**

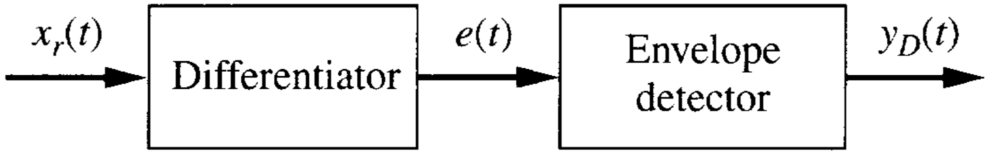
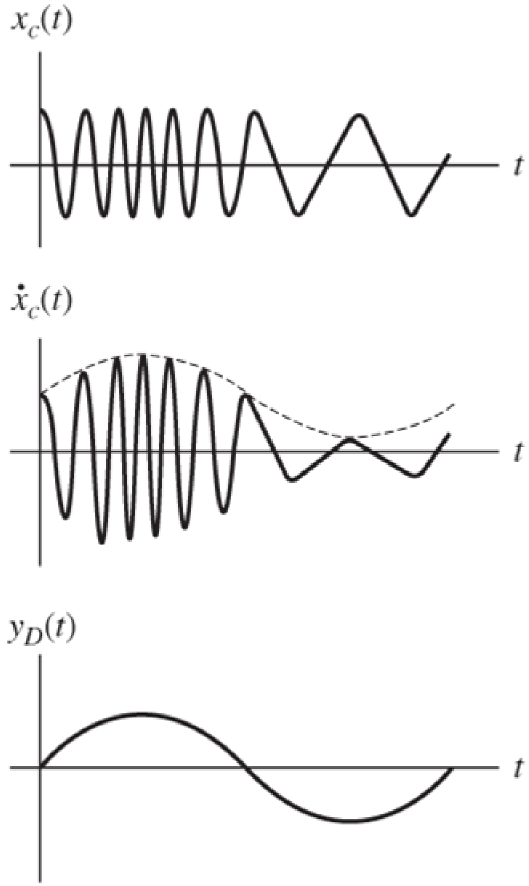


Figure 3.33
Frequency modulation discriminator.



$$e(t) = \frac{dx_r(t)}{dt} = -A_C(\omega_c + \frac{d\phi(t)}{dt})\sin(\omega_c t + \phi(t)).$$

$$y_D(t) = \text{output of envelope detector}$$

$$= A_c \frac{d\phi(t)}{dt} = 2\pi A_C f_d m(t).$$

(Carlson, Fig.5.3-7)

FM Demodulator **(2)**

- To reduce the channel noise effect, an amplitude *limiter* and a *BPF* are placed before the differentiator --- a **band-pass limiter**.

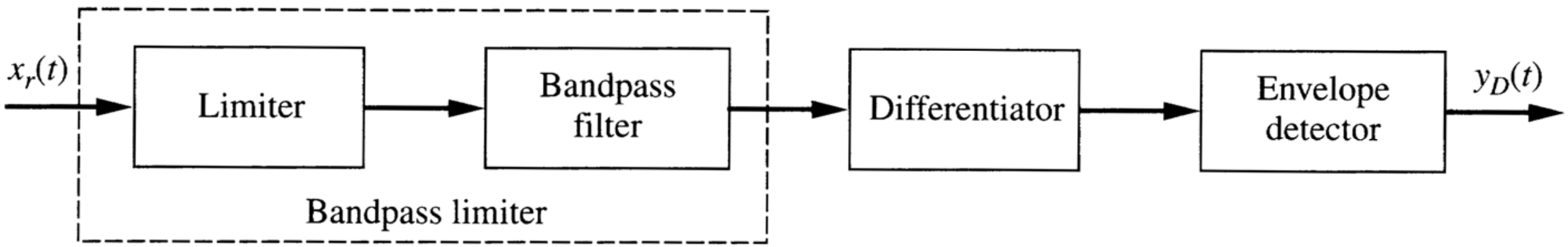


Figure 3.34
Frequency modulation discriminator with bandpass limiter.

Discriminator Implementation

- A “differentiation” operation

1) Time-delay:

$$e(t) = x_r(t) - x_r(t-\tau),$$

for small τ

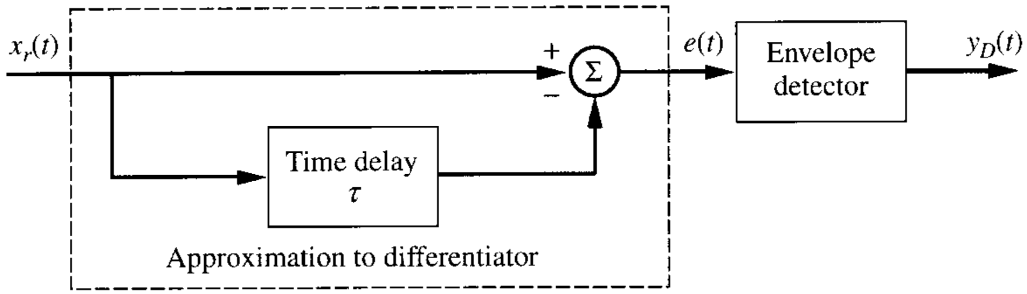


Figure 3.35
Discriminator implementation using delay and envelope detection.

2) RC network:

$$H(f) = \frac{R}{R + \dfrac{1}{j2\pi fC}} = \frac{j2\pi fRC}{1 + j2\pi fRC} \approx j2\pi fRC, \text{ for small } RC \ll 1$$

Disadvantage: Small K_D~RC → not practical

RC Network Differentiator

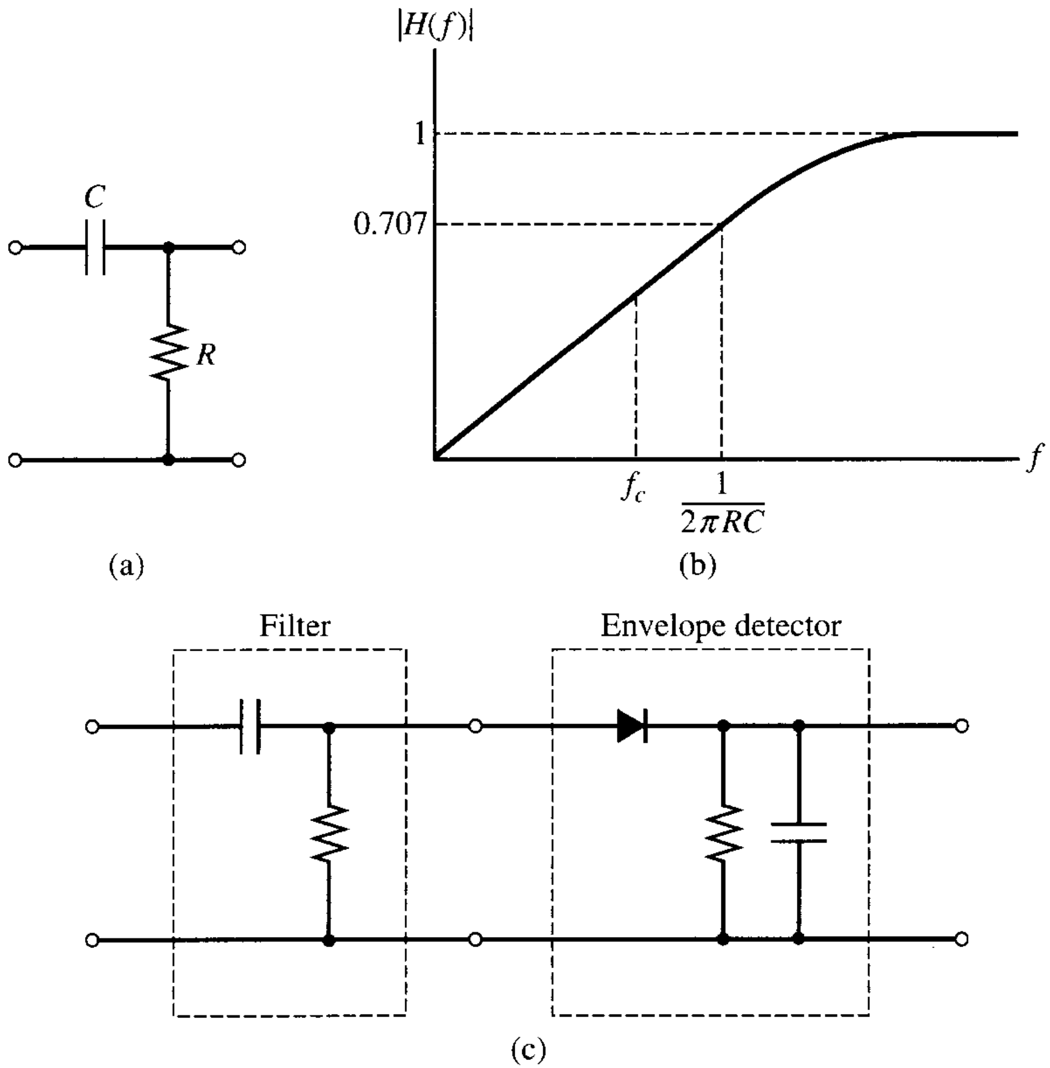


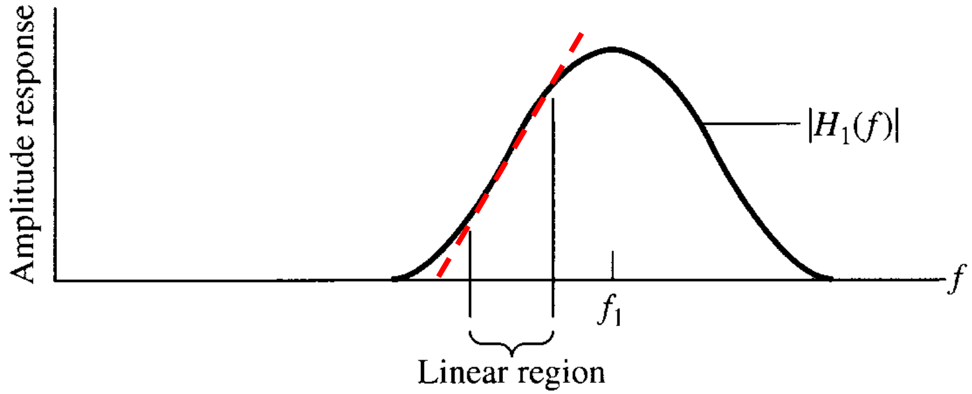
Figure 3.36
Implementation of a simple discriminator. (a) RC network. (b) Transfer function. (c) Simple discriminator.

Discriminator Implementation **(2)**

3) <u>BPF</u>

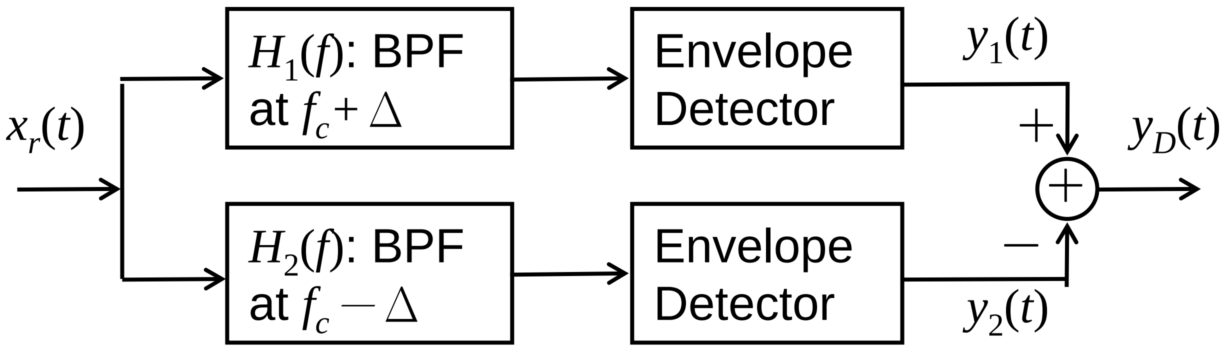
Disadvs: a) Small linear region

b) Has dc bias ($H(f)$ should b 0 at f=0)



4) **<u>Balanced discriminator</u>**: Two BPFs

$y_D(t)=y_1(t)-y_2(t)$
=envelope of IFT
$[X_r(f)(|H_1(f)|-|H_2(f)|)]$



Balanced Discriminator

- *Advs*:

  a) Wider linear range

  ($H(f)=|H_1(f)|-|H_2(f)|$)

  b) No dc bias ($H(f=f_c)=0$)