Principles of Communications Lecture 4: Analog Modulation Techniques (2)

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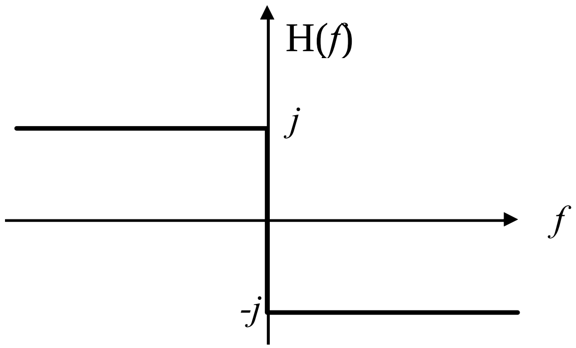
Outlines

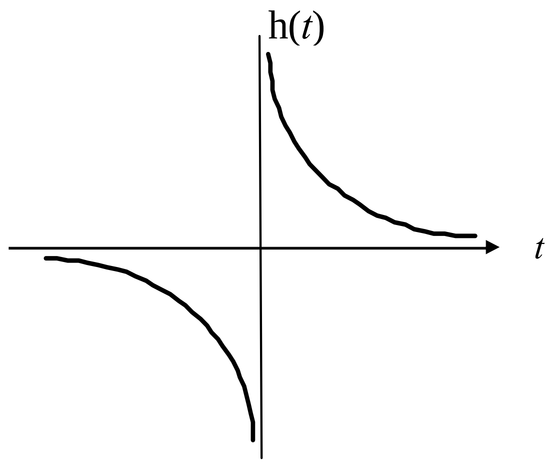
- Linear Modulation
- Angle Modulation
- Interference
- Feedback Demodulators
- Analog Pulse Modulation
- Delta Modulation and PCM
- Multiplexing

Hilbert Transform

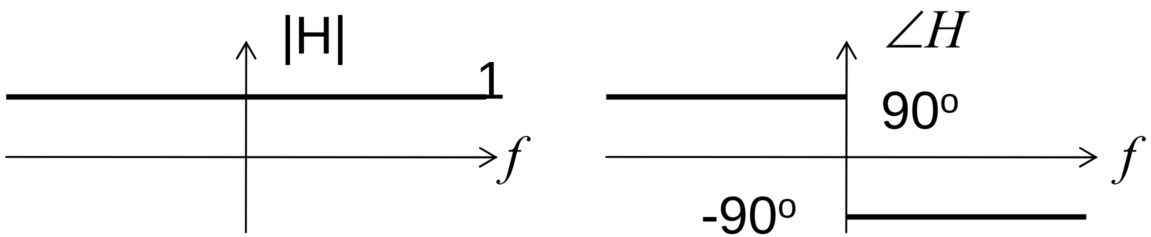
$$H(f) = -j\,\mathrm{sgn}(f),\ \mathrm{sgn}(f) = \begin{cases} 1 & f > 0 \\ 0 & f = 0 \\ -1 & f < 0 \end{cases}.$$



Impulse response: $h(t) = \dfrac{1}{\pi t}.$



- *Remark*: sgn()=signum function
- *Magnitude* and *phase*



Remarks

- Hilbert transform is just a system that simply phase-shifts all frequency components of its input by -½π
- $h(t) = \frac{1}{\pi t}$ Not abs-integrable, since the value is infinite at $t \to 0$
- Consider $G_\alpha(f) = \begin{cases} e^{-\alpha f}, & f > 0 \\ -e^{\alpha f}, & f < 0 \end{cases}$ Note: $\lim_{\alpha \to 0} G_\alpha(f) = \text{sgn}(f)$

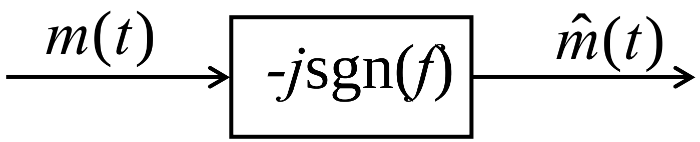
then $\mathfrak{I}^{-1}[G_\alpha(f)] = g(t;\alpha) = \int_0^\infty e^{-\alpha f} e^{j2\pi ft} df - \int_{-\infty}^0 e^{\alpha f} e^{j2\pi ft} df = \frac{j4\pi t}{\alpha^2 + (2\pi t)^2}$

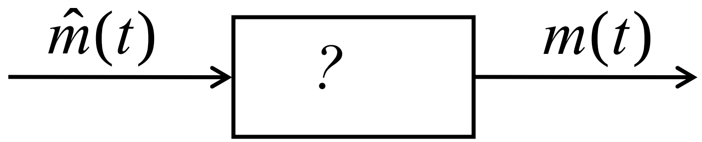
$$\Rightarrow \lim_{\alpha \to 0} \mathfrak{I}^{-1}[G_\alpha(f)] = \lim_{\alpha \to 0} g(\alpha;t)$$

$$= \lim_{\alpha \to 0} \frac{j4\pi}{\alpha^2 + (2\pi t)^2} = \frac{j}{\pi t}$$

Hilbert Transform (2)

- Hilbert transform is a liner system





$$\hat{x}(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau) \frac{1}{\pi(t-\tau)} d\tau$$

$$?(\text{inverse}) = -H(f)$$

$$\text{Pf}) \quad H(f)H(f) = -1$$

$$\text{Thus,} \quad \hat{\hat{x}}(t) = -x(t)$$

- *Ex*:

$$x(t) = \cos(\omega_0 t) \Rightarrow \hat{x}(t) = \sin(\omega_0 t).$$

$$x(t) = \sin(\omega_0 t) \Rightarrow \hat{x}(t) = -\cos(\omega_0 t).$$

$$x(t) = e^{j\omega_0 t} \Rightarrow \hat{x}(t) = -j\,\text{sgn}(\omega_0) e^{j\omega_0 t}. \quad \omega_0 > 0, \ldots$$

Properties of Hilbert Transform

(1) Energy (or power) of $x(t)$ = energy (or power) of $\hat{x}(t)$.

Pf) $|\hat{X}(f)| = \ldots = |X(f)|$

Check spectra density:

$$\left|\hat{X}(f)\right|^2 = \left|\Im[\hat{x}(t)]\right|^2 = \left|H(f)\right|^2 \left|X(f)\right|^2 = \left|-j\,\mathrm{sgn}(f)\right|^2 \left|X(f)\right|^2 = \left|X(f)\right|^2$$

(2) If $x(t)$ is real, then $x(t)$ and $\hat{x}(t)$ are **orthogonal**.

$$\int_{-\infty}^{\infty} x(t)\hat{x}(t)dt = 0 \quad \text{(for energy signals).}$$

$$\text{Or, } \lim_{T\to 0} \frac{1}{2T}\int_{-T}^{T} x(t)\hat{x}(t)dt = 0 \quad \text{(for power signals).}$$

(2) $\int_{-\infty}^{\infty} x(t)\hat{x}(t)dt = 0$

Pf) $\int_{-\infty}^{\infty} x(t)\hat{x}(t)dt = \int_{-\infty}^{\infty} X(f)\hat{X}^*(f)df$

(Generalized Parseval's Theorem)

$$\int_{-\infty}^{\infty} X(f)\hat{X}^*(f)df = \int_{-\infty}^{\infty} X(f)\, j\,\mathrm{sgn}(f) X^*(f)df$$

$$= j\int_{-\infty}^{\infty} \mathrm{sgn}(f)\,|X(f)|^2\,df$$

$$= j\left[\int_{-\infty}^{0}(-1)\,|X(f)|^2\,df + \int_{0}^{\infty}(1)\,|X(f)|^2\,df\right]$$

$|X(f)|^2$ is even $\Rightarrow$ ...=0

(3) If $m(t)$ is a **low-pass** signal and

$c(t)$ **high-pass**, and their spectra do not overlap,

then $H\{m(t)c(t)\} = m(t)\hat{c}(t)$,

where $H\{\cdot\}$ is Hilbert transform.

Pf) $m(t)c(t) = \Im^{-1}[M(f)]\Im^{-1}[C(f')]$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} M(f)C(f')e^{j2\pi(f+f')t}\,df\,df'$$

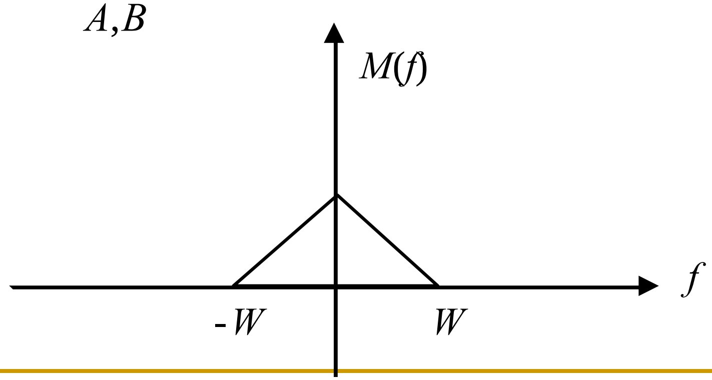
$$= \iint_{A,B} M(f)C(f')e^{j2\pi(f+f')t}\,df\,df'$$

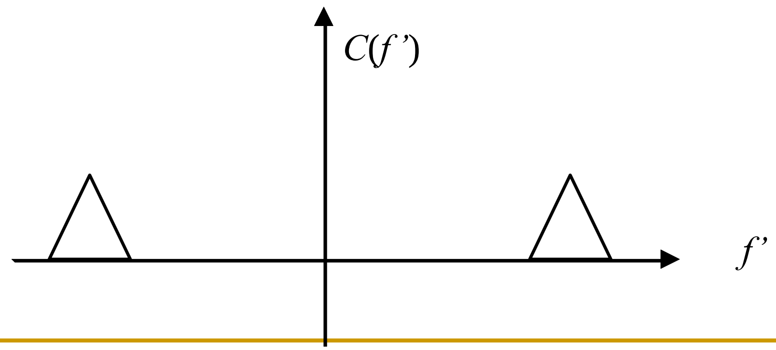
$Ex: c(t) = \cos\omega_0 t$,

$\omega_0 >$ BW of $M(f)$

$H\{m(t)c(t)\} = m(t)\hat{c}(t)$

$= m(t)\sin\omega_0 t$





$$H\{m(t)c(t)\} = m(t)c(t) * \left(\frac{1}{\pi t}\right)$$

$$= \iint_{A,B} M(f)C(f')\left[e^{j2\pi(f+f')t} * \frac{1}{\pi t}\right] df df'$$

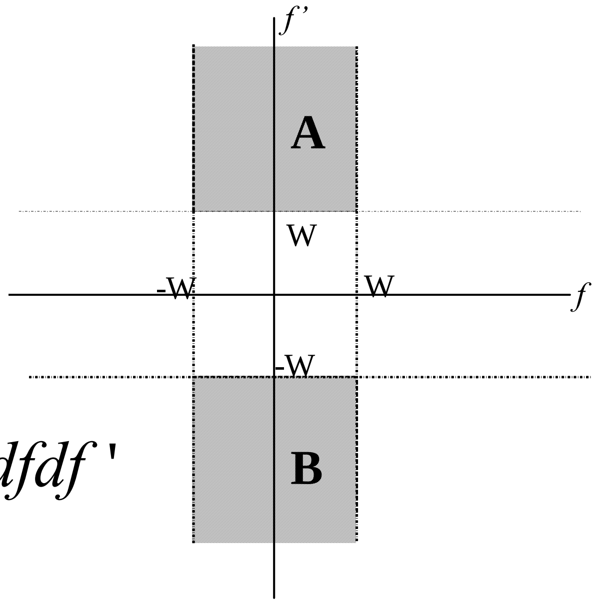
$$= \iint_{A,B} M(f)C(f')[-j\,\mathrm{sgn}(f+f')]e^{j2\pi(f+f')t} df df'$$

$$= \iint_{A,B} M(f)C(f')[-j\,\mathrm{sgn}(f')]e^{j2\pi(f+f')t} df df'$$

$$= \iint_{A,B} M(f)e^{j2\pi ft}C(f')[-j\,\mathrm{sgn}(f')]e^{j2\pi f't} df df'$$

$$= \int_{-\infty}^{\infty} M(f)e^{j2\pi ft} df \int_{-\infty}^{\infty} C(f')[-j\,\mathrm{sgn}(f')]e^{j2\pi f't} df'$$

$$= m(t)\hat{c}(t)$$



Regions A&B: M(f) and C(f) have nonzero values

- $H\{e^{j(\omega+\omega')t}\} = -j\,\mathrm{sgn}(\omega+\omega')\,e^{j(\omega+\omega')t}$.
-

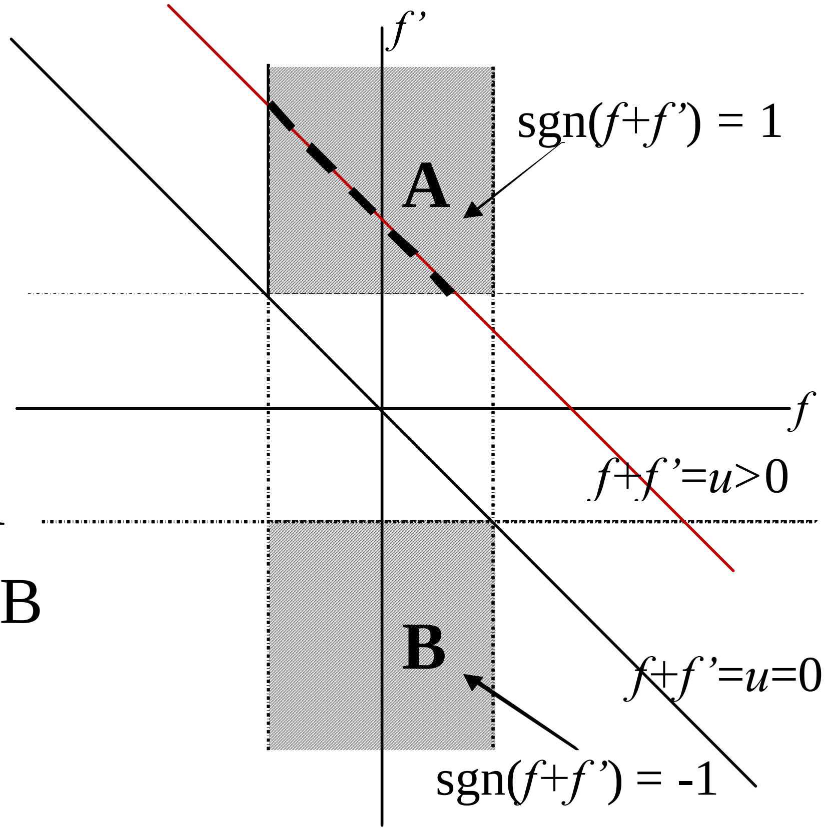
$\mathrm{sgn}(f+f') = ?\,\mathrm{sgn}(f')$

Let $f+f'=u$

$u=0 \rightarrow f+f'=0 \rightarrow$ no intersection

$u>0 \rightarrow \mathrm{sgn}(f+f')=1=\mathrm{sgn}(f')$ in A

$u<0 \rightarrow \mathrm{sgn}(f+f')=-1=\mathrm{sgn}(f')$ in B



Analytic Signals

$$\begin{cases} \text{If } x(t) \text{ is real} \underset{\substack{\text{analytic}\\ \text{signal}}}{\longrightarrow} \text{Define } x_P(t) = x(t) + j\hat{x}(t). \\ x(t) = \text{Re}\{x_P(t)\} = \frac{1}{2}\{x_P(t) + x_P^*(t)\}. \end{cases}$$

- Key: If $x(t)$ is a low-pass signal

  → $x_p(t)$ has only *one-sided* spectrum
- Usage: (1) SSB signal

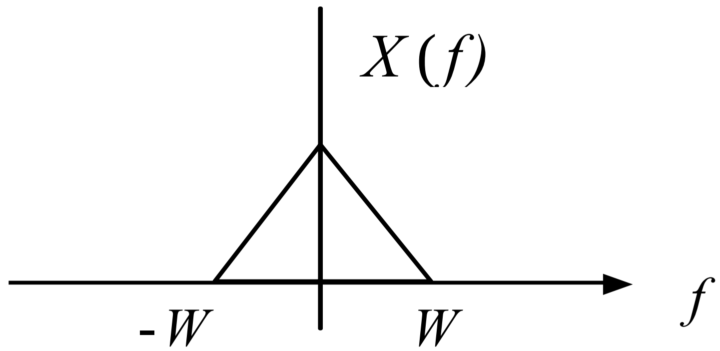
  (2) Complex envelopes of band-pass signals

- $$X_p(f) = \begin{cases} 2X(f), & f > 0 \\ 0, & f < 0 \end{cases}$$

Pf) $x_P(t) = x(t) + j\hat{x}(t)$

$$FT \Rightarrow Xp(f) = X(f) + j(-j\,\mathrm{sgn}(f)X(f))$$

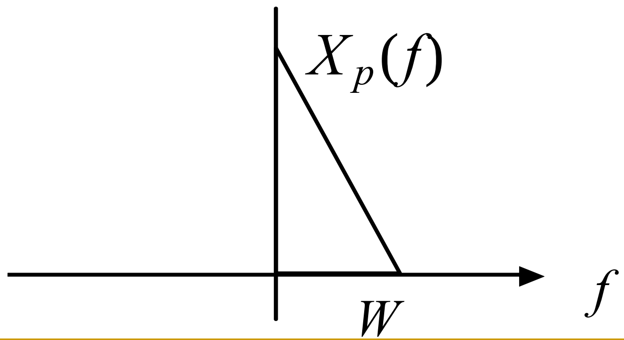
$$= X(f)\left[1 + \mathrm{sgn}(f)\right]$$



- Similarly,

$$x_n(t) = x(t) - j\hat{x}(t)$$

$$X_n(f) = \begin{cases} 0, & f > 0 \\ 2X(f), & f < 0 \end{cases}$$



Band-pass Signal

- BP signal: centered around f_0 in freq domain

$$x(t) = a(t)\cos[2\pi f_0 t + \theta(t)]$$
$$= x_R(t)\cos 2\pi f_0 t - x_I(t)\sin 2\pi f_0 t$$

in-phase quadrature

$$\begin{cases} a(t) = \sqrt{x_R^2(t) + x_I^2(t)} \\ \theta(t) = \tan^{-1}\left(\dfrac{x_I(t)}{x_R(t)}\right) \end{cases}$$

$a(t)$: (natural) envelope.

Equivalent Bandpass Signal Representation

- Bandpass signal:

❶ $x(t) = a(t)\cos\left(2\pi f_0 t + \theta(t)\right)$

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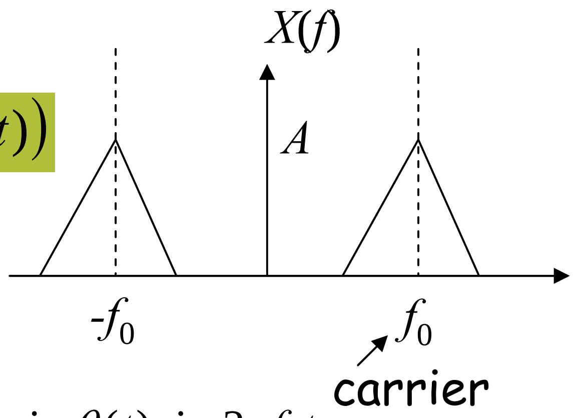
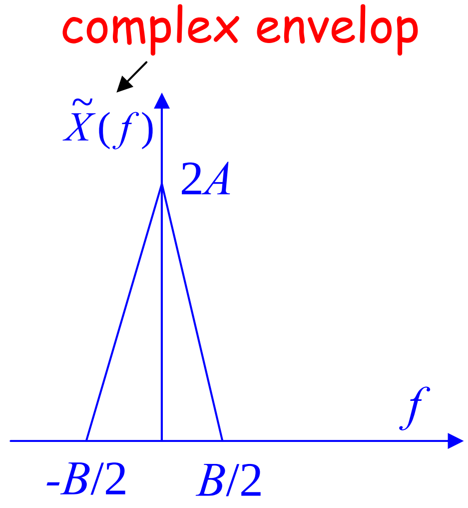




$$x(t) = a(t)\cos\left(2\pi f_0 t + \theta(t)\right)$$
$$= a(t)\cos\theta(t)\cos 2\pi f_0 t - a(t)\sin\theta(t)\sin 2\pi f_0 t$$

❷ $\equiv x_R(t)\cos 2\pi f_0 t - x_I(t)\sin 2\pi f_0 t$

inphase quadrature zero-frequency parts of $x(t)$, lowpass signals

where
$$\begin{cases} a(t) = \sqrt{x_R^2(t) + x_I^2(t)} \\ \theta(t) = \tan^{-1}\left(\dfrac{x_I(t)}{x_R(t)}\right) \end{cases}$$

❸ $x(t) = \mathrm{Re}\{a(t)e^{j2\pi f_0 t + \theta(t)}\} = \mathrm{Re}\{\tilde{x}(t)e^{j2\pi f_0 t}\}$

where $\tilde{x}(t)$ is called the complex envelop of $x(t)$

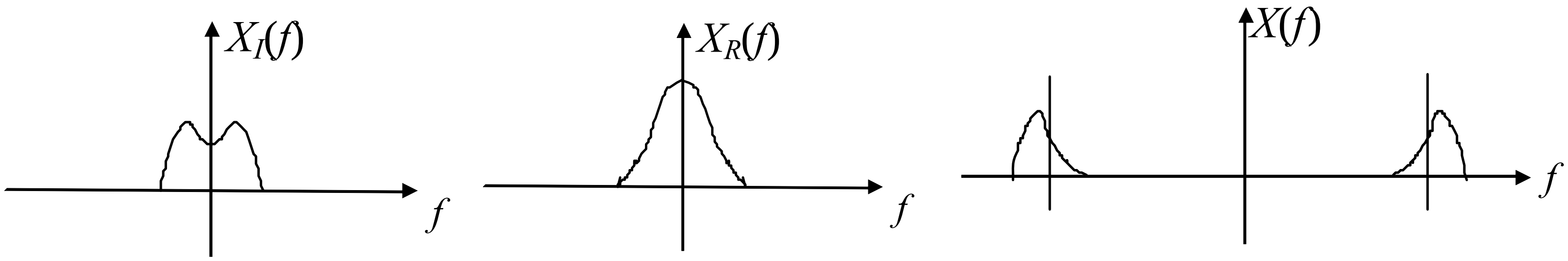
$$\tilde{x}(t) = a(t)e^{j\theta(t)} = x_R(t) + jx_I(t)$$

BP Signals (2)

- *Remarks*: (1) In general, $a(t)$ and $\theta(t)$ can be two independent time-varying signals. If we can extract them separately without interferences, we can send two independent messages.

(2) Two independent messages can also be in the form of $x_R(t)$ and $x_I(t)$. They are carried by two orthogonal carriers, cos() and sin().

- *Q*: Can we remove the carrier of a BP signal, and obtain an ***equivalent*** *baseband* signal?



Complex Envelope

- Def: **Complex envelope**

$$\tilde{x}(t) \equiv x_R(t) + jx_I(t) = x_P(t)e^{-j2\pi f_0 t}$$

- If we start with a real baseband signal, the corresponding BP signal has *amplitude* information only (one message).
- If we start with a real BP signal, the corresponding baseband signal becomes *complex* (two messages).
- The *complex envelope* of an arbitrary BP signal is an equivalent expression that contains two messages.

Complex Envelope (2)

- How to convert a BP signal $x(t)$ to its complex envelope $\tilde{x}(t)$ mathematically?

$$\begin{matrix} x_R(t) \\ x_I(t) \end{matrix} \leftrightarrow \tilde{x}(t) \underset{e^{-j2\pi f_0 t}}{\overset{e^{j2\pi f_0 t}}{\rightleftarrows}} x_P(t) \underset{x(t)+j\hat{x}(t)}{\overset{\mathrm{Re}}{\rightleftarrows}} x(t)$$

real complex complex real

- *Remarks*: $\tilde{x}(t)$ and $x_p(t)$ do NOT exist in physical world. They are mathematical models, convenient in mathematical operation.

To show $x(t) \rightarrow x_p(t) \rightarrow \tilde{x}(t)$

(1) Start with $x(t) = x_R(t)\cos 2\pi f_0 t - x_I(t)\sin 2\pi f_0 t$.

$$\hat{x}(t) = x_R(t)\sin 2\pi f_0 t - x_I(t)(-\cos 2\pi f_0 t)$$

$$= x_R(t)\sin 2\pi f_0 t + x_I(t)\cos 2\pi f_0 t.$$

(Assume $x_R(t)$ and $x_I(t)$ baseband signals and use Hilber transform Property 3.)

(2) $x_P(t) = x(t) + j\hat{x}(t)$

$$= x_R(t)[\cos 2\pi f_0 t + j\sin 2\pi f_0 t]$$

$$+ x_I(t)[j\cos 2\pi f_0 t - \sin 2\pi f_0 t]$$

$$= x_R(t)e^{j2\pi f_0 t} + jx_I(t)e^{j2\pi f_0 t}$$

$$= [x_R(t) + jx_I(t)]e^{j2\pi f_0 t} = \tilde{x}(t)e^{j2\pi f_0 t}.$$

Therefore, $\tilde{x}(t) = x_P(t)e^{-j2\pi f_0 t}$.

Summary

- Analytic signal: signal has only positive spectrum

$$x_P(t) = x(t) + j\hat{x}(t)$$

- Complex envelop: shift analytic signal to the "baseband".

- Real signal is completely specified by analytic signal but complex signal may not (not symmetric in freq)
- The complex envelop of a BP DSB signal is simply the message itself.

Illustrations of $x(t)$, $x_p(t)$, and $\tilde{x}(t)$

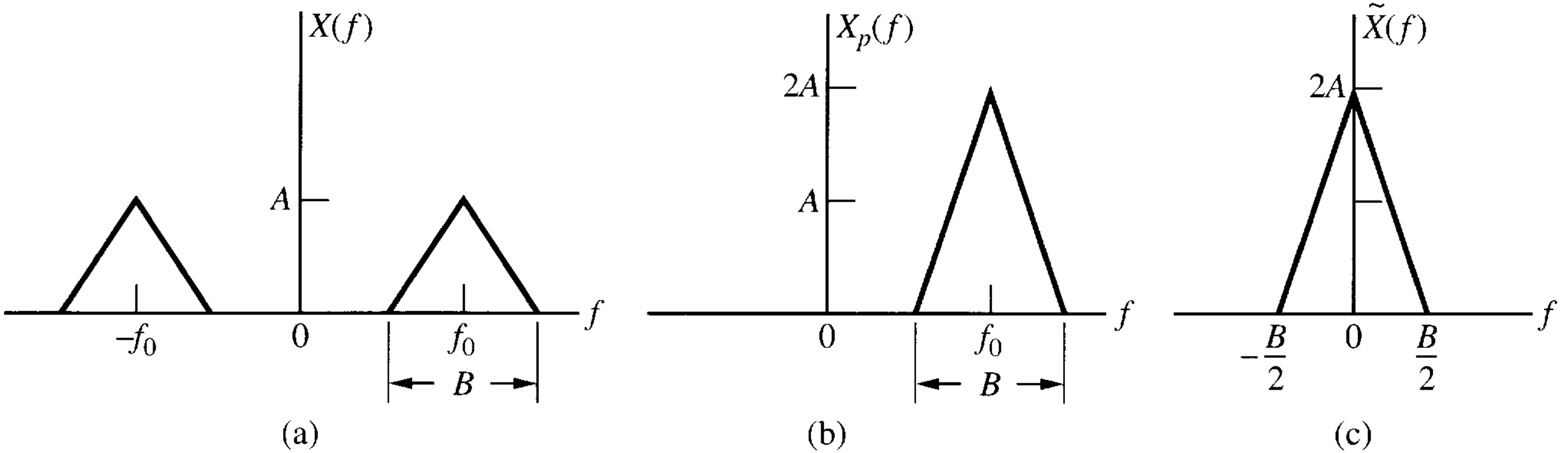


Figure 2.31

Spectra pertaining to the formation of a complex envelope of a signal $x(t)$. (a) A bandpass signal spectrum. (b) Twice the positive-frequency portion of $X(f)$ corresponding to $\Im[x(t) + j\,\hat{x}(t)]$. (c) Spectrum of $\tilde{x}(t)$.

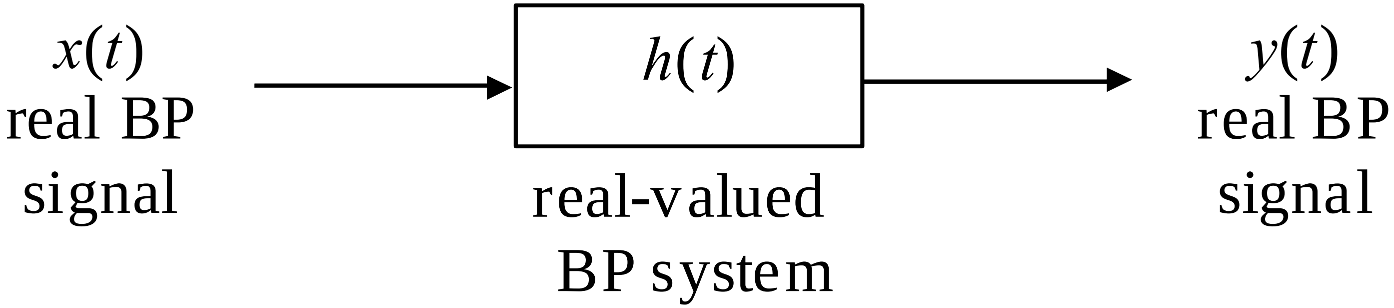
Bandpass Systems

- A physical system in passband (e.g., a channel) may have different *in-phase* and *quadrature* impulse responses.
- If the BP impulse response is $h(t)$, its *equivalent baseband* complex envelope (of this BP system) is:

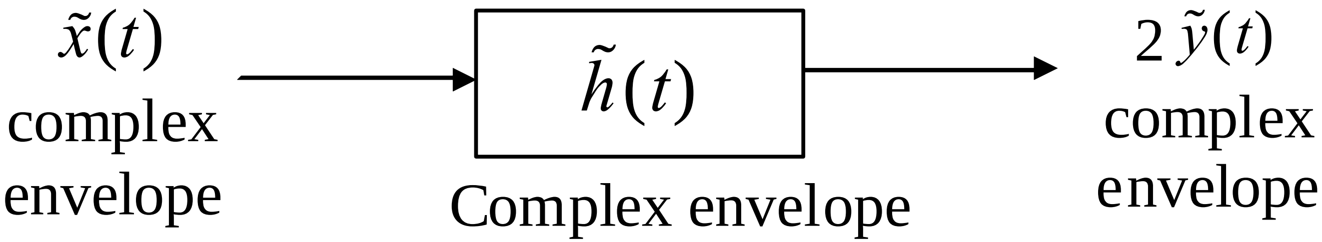
$$\tilde{h}(t) \equiv h_R(t) + jh_I(t)$$

BP Systems (2)

- Physical (*real-valued*) BP system



- Math model (equivalent *complex-valued* representation)



$$2\tilde{y}(t) = \tilde{h}(t) * \tilde{x}(t)$$

Proof:

$$h(t) = \mathrm{Re}\{\tilde{h}(t)e^{j2\pi f_0 t}\} = \frac{1}{2}\tilde{h}(t)e^{j2\pi f_0 t} + \frac{1}{2}[\tilde{h}(t)e^{j2\pi f_0 t}]^* = \frac{1}{2}\tilde{h}(t)e^{j2\pi f_0 t} + \frac{1}{2}\tilde{h}^*(t)e^{-j2\pi f_0 t}.$$

$$x(t) = \mathrm{Re}\{\tilde{x}(t)e^{j2\pi f_0 t}\} = \frac{1}{2}\tilde{x}(t)e^{j2\pi f_0 t} + \frac{1}{2}\tilde{x}^*(t)e^{-j2\pi f_0 t}.$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda$$

$$= \int_{-\infty}^{\infty} [\frac{1}{2}\tilde{h}(\lambda)e^{j2\pi f_0 \lambda} + \frac{1}{2}\tilde{h}^*(\lambda)e^{-j2\pi f_0 \lambda}] \cdot [\frac{1}{2}\tilde{x}(t-\lambda)e^{j2\pi f_0 (t-\lambda)} + \frac{1}{2}\tilde{x}^*(t-\lambda)e^{-j2\pi f_0 (t-\lambda)}]d\lambda$$

$$= \frac{1}{4}\int_{-\infty}^{\infty} \tilde{h}(\lambda)\tilde{x}(t-\lambda)e^{j2\pi f_0 t}d\lambda + \frac{1}{4}\int_{-\infty}^{\infty} \tilde{h}^*(\lambda)\tilde{x}^*(t-\lambda)e^{-j2\pi f_0 t}d\lambda$$

$$+ \frac{1}{4}\int_{-\infty}^{\infty} \tilde{h}(\lambda)\tilde{x}^*(t-\lambda)e^{j4\pi f_0 \lambda}d\lambda \cdot e^{-j2\pi f_0 t} + \frac{1}{4}\int_{-\infty}^{\infty} \tilde{h}^*(\lambda)\tilde{x}(t-\lambda)e^{-j4\pi f_0 \lambda}d\lambda \cdot e^{j2\pi f_0 t}.$$

$$\because \quad \int_{-\infty}^{\infty} \tilde{h}(\lambda)\tilde{x}^*(t-\lambda)e^{j4\pi f_0 \lambda}d\lambda = \tilde{h}(t)e^{j4\pi f_0 t} * \tilde{x}^*(t) = \mathfrak{I}^{-1}\{\tilde{H}(f-2f_0)\tilde{X}^*(-f)\} = 0.$$

No overlap in freq domain

$$y(t) = \frac{1}{4}\int_{-\infty}^{\infty} \tilde{h}(\lambda)\tilde{x}(t-\lambda)e^{j2\pi f_0 t}d\lambda + \frac{1}{4}\int_{-\infty}^{\infty} \tilde{h}^*(\lambda)\tilde{x}^*(t-\lambda)e^{-j2\pi f_0 t}d\lambda$$

$$= \frac{1}{4}(\tilde{h}(t) * \tilde{x}(t)) \cdot e^{j2\pi f_0 t} + \frac{1}{4}(\tilde{h}^*(t) * \tilde{x}^*(t)) \cdot e^{-j2\pi f_0 t}$$

$$= \frac{1}{2}\mathrm{Re}\{(\tilde{h}(t) * \tilde{x}(t)) \cdot e^{j2\pi f_0 t}\} = \mathrm{Re}\{\tilde{y}(t)e^{j2\pi f_0 t}\}, \text{ if } \tilde{h}(t) * \tilde{x}(t) = 2\tilde{y}(t).$$

Concluding Remarks

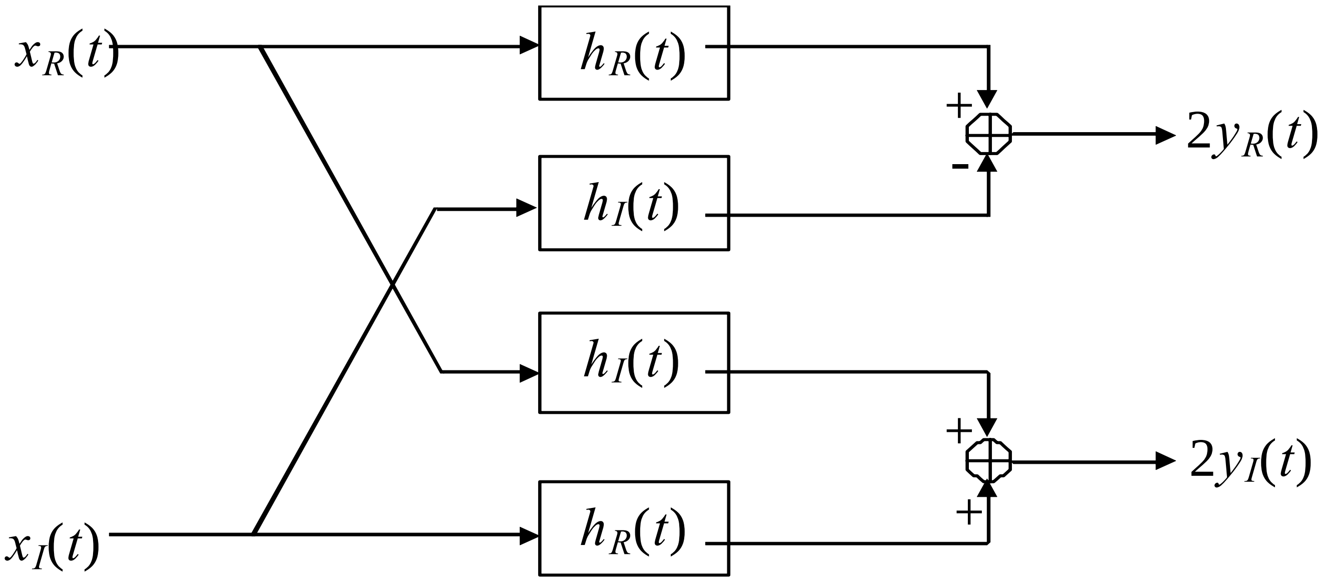
- On p.90, Z&T, $y(t) = \frac{1}{2}\mathrm{Re}\{(\tilde{h}(t) * \tilde{x}(t)) \cdot e^{j2\pi f_0 t}\}$

$$= \frac{1}{2}\mathrm{Re}\{\tilde{y}(t)e^{j2\pi f_0 t}\}$$

- On p.88 (eq 2.314): $x(t) = \mathrm{Re}\{\tilde{x}(t)e^{j2\pi f_0 t}\}$ without ½.
- On p.89 (eq 2.324): $h(t) = \mathrm{Re}\{\tilde{h}(t)e^{j2\pi f_0 t}\}$ without ½.
  - If $\tilde{y}(t)$ is defined without ½, then $2\tilde{y}(t) = \tilde{h}(t) * \tilde{x}(t)$
  - If we follow the definition of Z&T on $\tilde{y}(t)$, then $\tilde{y}(t) = \tilde{h}(t) * \tilde{x}(t)$

- Either way is fine.

BP System Implementation



$$h(t) = h_R(t)\cos 2\pi f_0 t - h_I(t)\sin 2\pi f_0 t$$

$$x(t) = x_R(t)\cos 2\pi f_0 t - x_I(t)\sin 2\pi f_0 t$$

$$y(t) = y_R(t)\cos 2\pi f_0 t - y_I(t)\sin 2\pi f_0 t$$

$$2\tilde{y}(t) = \tilde{h}(t) * \tilde{x}(t)$$

From Analytic Signal to SSB Signal

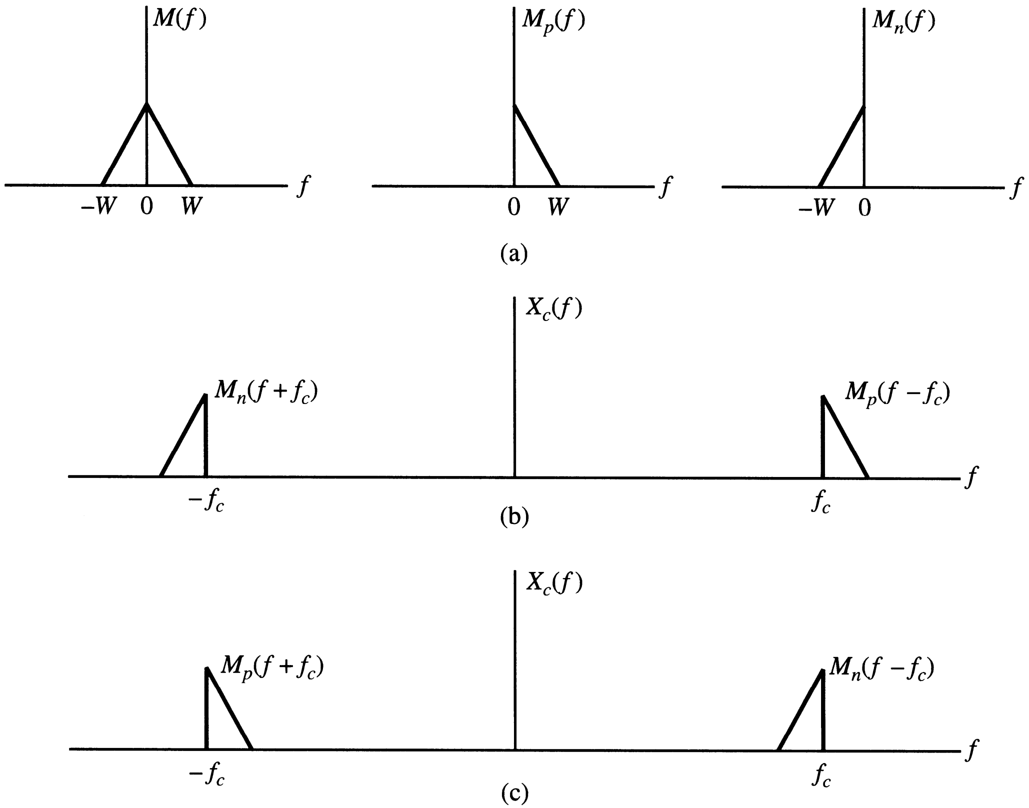
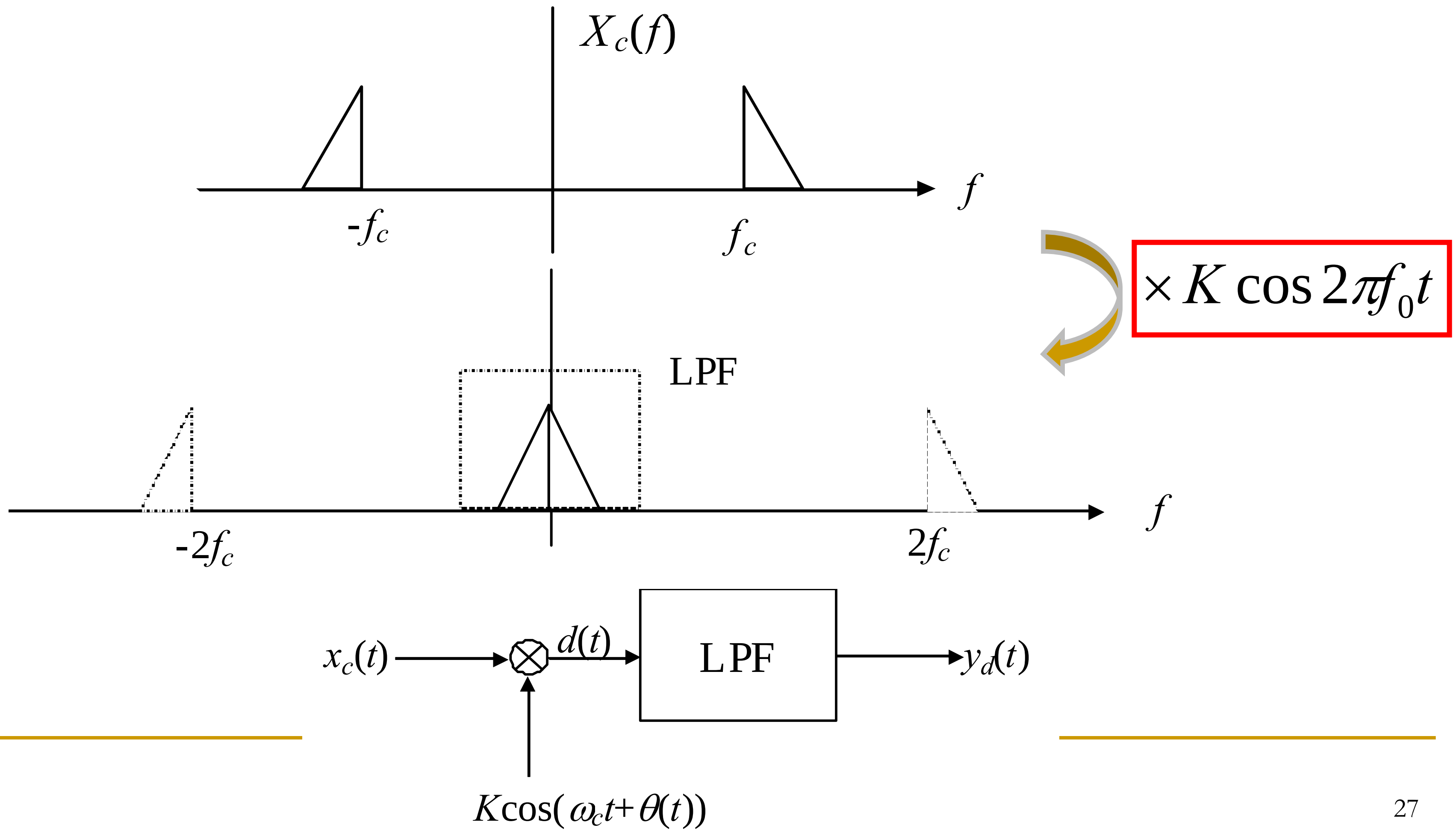


FIGURE 3.9 Alternative derivation of SSB signals. (a) $M(f)$, $M_p(f)$, and $M_n(f)$. (b) Upper-sideband SSB signal. (c) Lower-sideband SSB signal.

- $M_p(f)$: the positive-frequency portion of $M(f)$
- $M_n(f)$: the negative-frequency portion of $M(f)$
- Apply the frequency-translation theorem to both the $M_p(f)$ and $M_n(f)$
  - We obtain the upper-sideband SSB signal and lower-sideband SSB signal, respectively

SSB Demodulation

- **Coherent** detection with possibly *phase error*



Let $K = 4$, we have $d(t) = x_c(t) \cdot 4\cos(\omega_c t + \theta(t))$

$$= \frac{A_C}{2}[m(t)\cos\omega_c t + \hat{m}(t)\sin\omega_c t] \cdot 4\cos(\omega_c t + \theta(t))$$

$$= A_C m(t)\cos\theta(t) + A_C m(t)\cos[2\omega_c t + \theta(t)]$$

$$- A_C \hat{m}(t)\sin\theta(t) + A_C \hat{m}(t)\sin[2\omega_c t + \theta(t)].$$

After LPF, we have

LSB: $y_D(t) = m(t)\cos\theta(t) - \hat{m}(t)\sin\theta(t)$.

message cross-talk

USB: $y_D(t) = m(t)\cos\theta(t) + \hat{m}(t)\sin\theta(t)$.

Remarks: If there exist frequency and phase error in the local carrier, then

LSB: $y_D(t) = m(t)\cos[(\Delta f \cdot 2\pi)t + \theta(t)] - \hat{m}(t)\sin[(\Delta f \cdot 2\pi)t + \theta(t)]$.

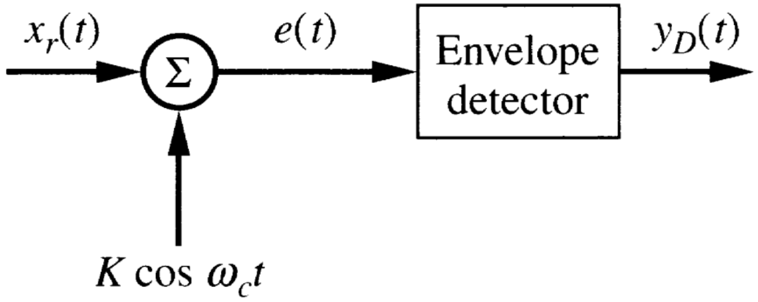
USB: $y_D(t) = m(t)\cos[(\Delta f \cdot 2\pi)t + \theta(t)] + \hat{m}(t)\sin[(\Delta f \cdot 2\pi)t + \theta(t)]$.

- **Carrier insertion** with envelope detector

$$e(t) = \frac{A_C}{2}[m(t)\cos\omega_c t \pm \hat{m}(t)\sin\omega_c t] + K\cos\omega_c t$$

$$= [\frac{A_C}{2}m(t) + K]\cos\omega_c t \pm \frac{A_C}{2}\hat{m}(t)\sin\omega_c t$$

$$\equiv a(t)\cos\omega_c t \pm b(t)\sin\omega_c t.$$



Define $\begin{cases} R(t) = \sqrt{a^2(t) + b^2(t)} \\ \theta(t) = \tan^{-1}\dfrac{b(t)}{a(t)} \end{cases}.$

$$\Rightarrow e(t) = R(t)\cos\theta(t)\cos\omega_c t \pm R(t)\sin\theta(t)\sin\omega_c t.$$

$$\Rightarrow y_D(t) = R(t) = \sqrt{\left[\frac{A_C}{2}m(t) + K\right]^2 + \left[\frac{A_C}{2}\hat{m}(t)\right]^2}.$$

If K is large enough, then $y_D(t) \approx \frac{A_C}{2}m(t) + K.$

Time-Domain SSB Waveform

- The SSB waveform is complicated in general

$$\begin{cases} m(t) = \cos\omega_1 t - 0.4\cos 2\omega_1 t + 0.9\cos 3\omega_1 t \\ \hat{m}(t) = \sin\omega_1 t - 0.4\sin 2\omega_1 t + 0.9\sin 3\omega_1 t \end{cases}$$

$$\rightarrow x_c(t) = \frac{A_C}{2}\left[m(t)\cos\omega_c t \pm \hat{m}(t)\sin\omega_c t\right]$$

$$= R(t)\cos\left(\omega_c t + \theta(t)\right)$$

$$\text{where } \begin{cases} R(t) = \frac{A_C}{2}\sqrt{m^2(t) + \hat{m}^2(t)} \\ \theta(t) = \pm\tan^{-1}\frac{\hat{m}(t)}{m(t)} \end{cases}.$$

SSB Time Signals

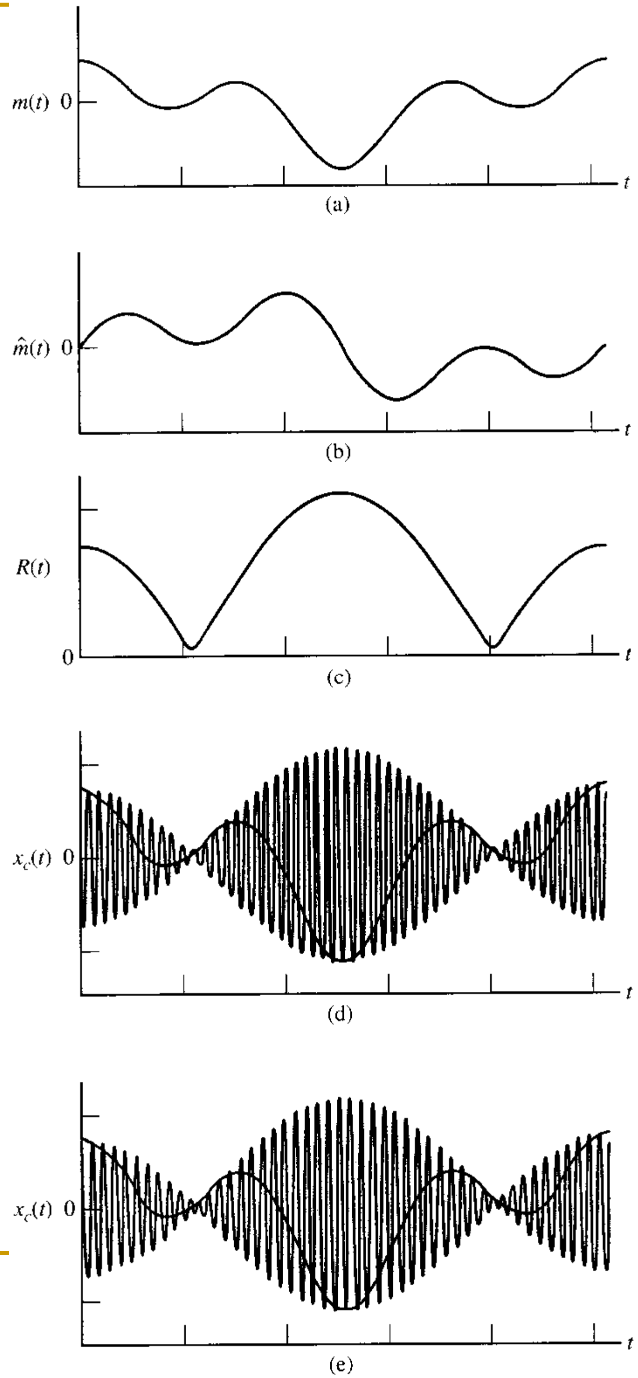


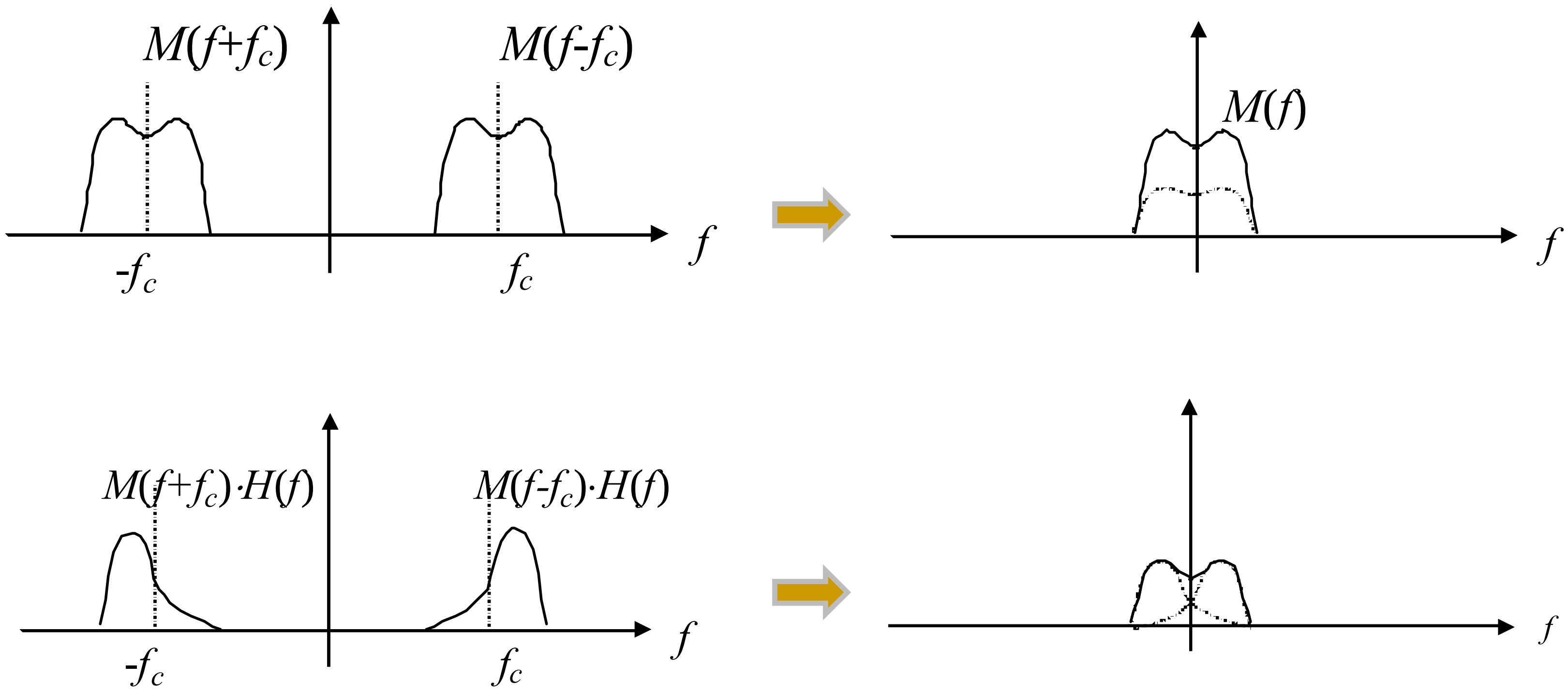
Figure 3.12
Time-domain signals for SSB system. (a) Message signal. (b) Hilbert transform of message signal. (c) Envelope of SSB signal. (d) Upper-sideband SSB signal with message signal. (e) Lower-sideband SSB signal with message signal.

Vestigial-SideBand Modulation (VSB)

- SSB has advantages over DSB and AM.
- *Problems* related to implementation of practical SSB systems:
  - (1) imperfect Hilbert filter (sharp cutoff),
  - (2) poor low-frequency components, and
  - (3) loss of carrier.
- Solutions? An intermediate scheme between DSB and SSB, i.e., do not cut out half of the spectrum completely. We call the new scheme **VSB**.

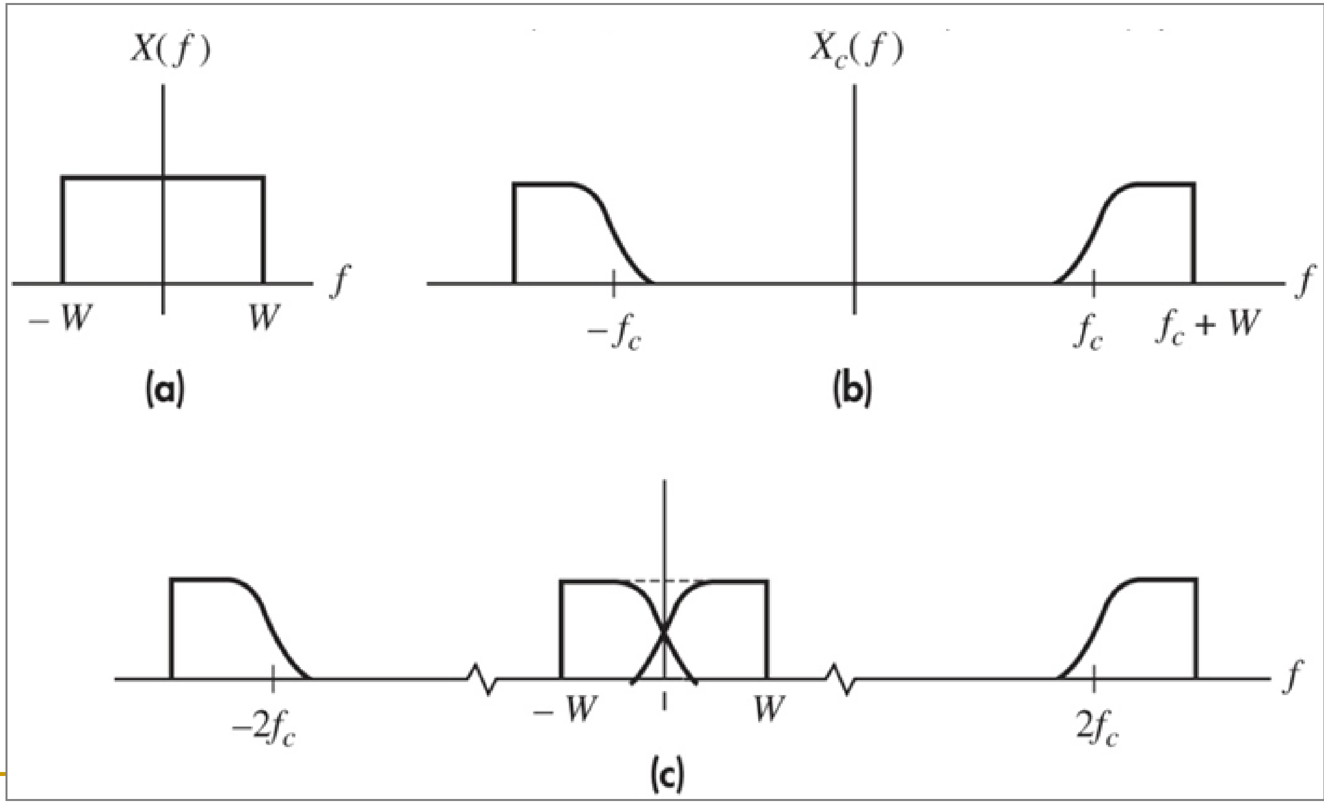
DSB vs. VSB Spectra

- <u>Concept</u>: Include a small amount of the other sideband – "symmetric filter".



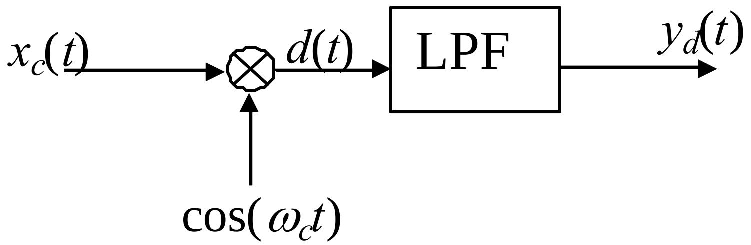
Requirements on H(f)

- VSB filter (Carlson, Fig4.4-8)
- Coherent Demod (Carlson, Fig4.5-4)



VSB Demodulation

- Coherent Detection



$$X_c(f) = [M(f+f_c) + M(f-f_c)]\cdot H(f) \quad \text{(VSB signal)}$$

$$D(f) = \Im\{d(t)\}$$

$$= \frac{1}{2}[M(f+2f_c) + M(f)]H(f+f_c) + \frac{1}{2}[M(f) + M(f-2f_c)]H(f-f_c)$$

$$= \frac{1}{2}M(f)[H(f+f_c) + H(f-f_c)]$$

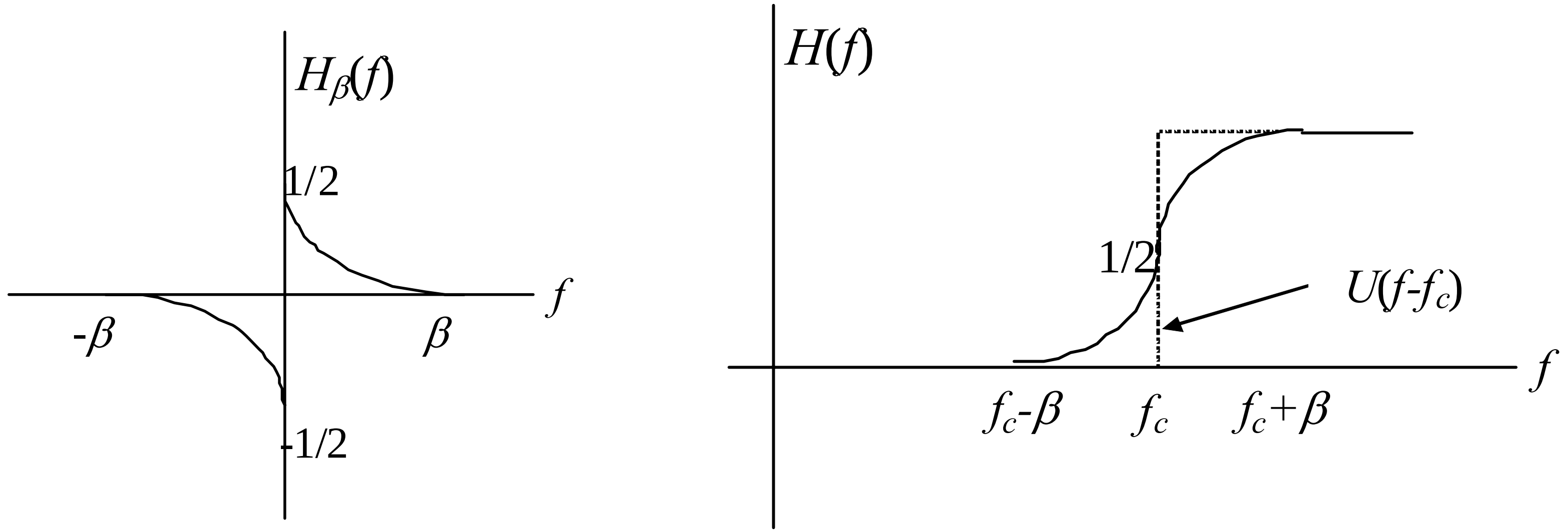
$$+\frac{1}{2}M(f+2f_c)H(f+f_c) + \frac{1}{2}M(f-2f_c)H(f-f_c).$$

After LPF,

$$Y_D(f) = \frac{1}{2}M(f)[H(f+f_c) + H(f-f_c)]\big|_{LP}.$$

- One solution: $H(f)$ is *mean-shifted conjugate anti-symmetric* about f_c. Explicitly, let $H_\beta(f)$ be an LP anti-symmetric filter; i.e., $H_\beta(f) = -H_\beta(-f)$ and $H_\beta(f) = 0$ for $|f| > \beta$.

$$H(f) = \begin{cases} U(f - f_c) - H_\beta(f - f_c) & \text{for } f > 0 \\ U(-f - f_c) + H_\beta(f + f_c) & \text{for } f < 0 \end{cases}$$



Time-domain Signal of VSB

- Often, carrier is added to transmitted signal. This is similar to SSB with carrier insertion. We only need to use an envelope detector at the receiver.

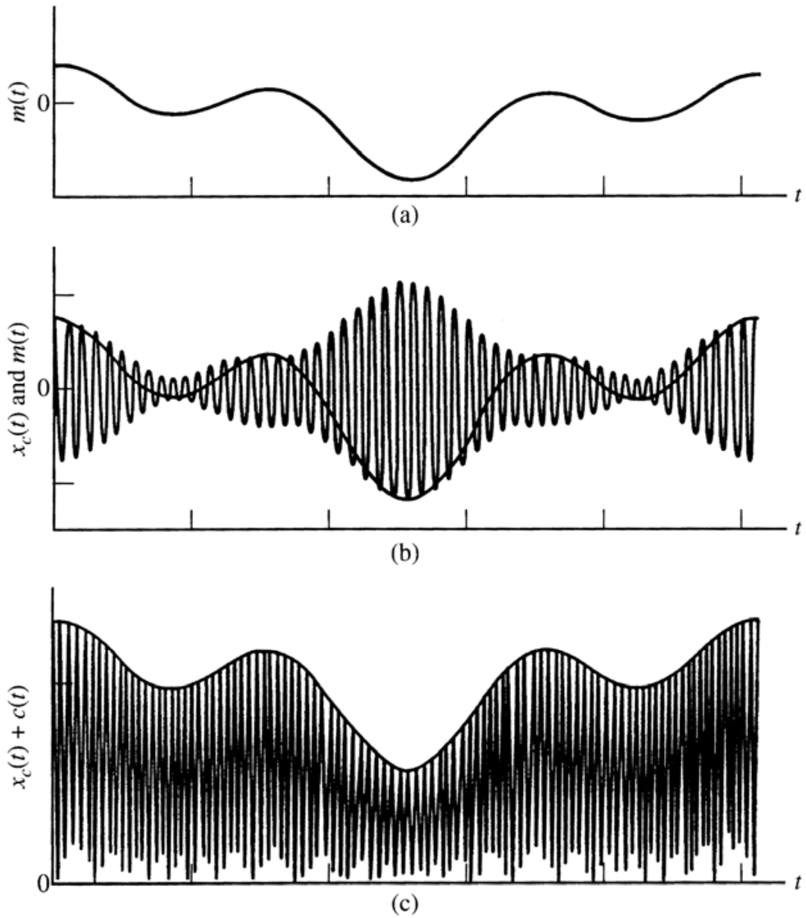


Figure 3.14
Time-domain signals for VSB system. (a) Message signal. (b) VSB signal and message signal. (c) Sum of VSB signal and carrier signal.

Transmitted TV Spectrum

- <u>Example</u>: Color TV; Video: VSB; Chrominance: I/Q; Audio: FM

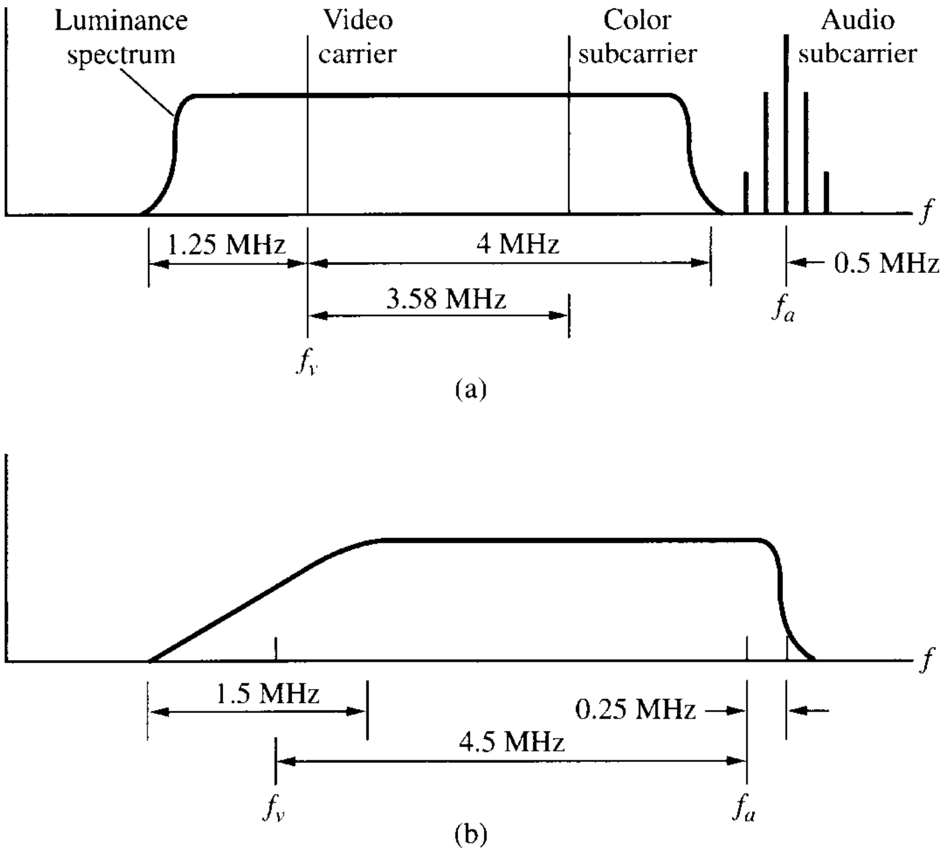


Figure 3.15
Transmitted spectrum and VSB filtering for television. (a) Spectrum of transmitted signal. (b) VSB filter in receiver.

Frequency Translation

- Goal: Translate a BP signal to a new carrier freq.
- **Mixing**: The process of multiplying a BP signal by a periodic signal.

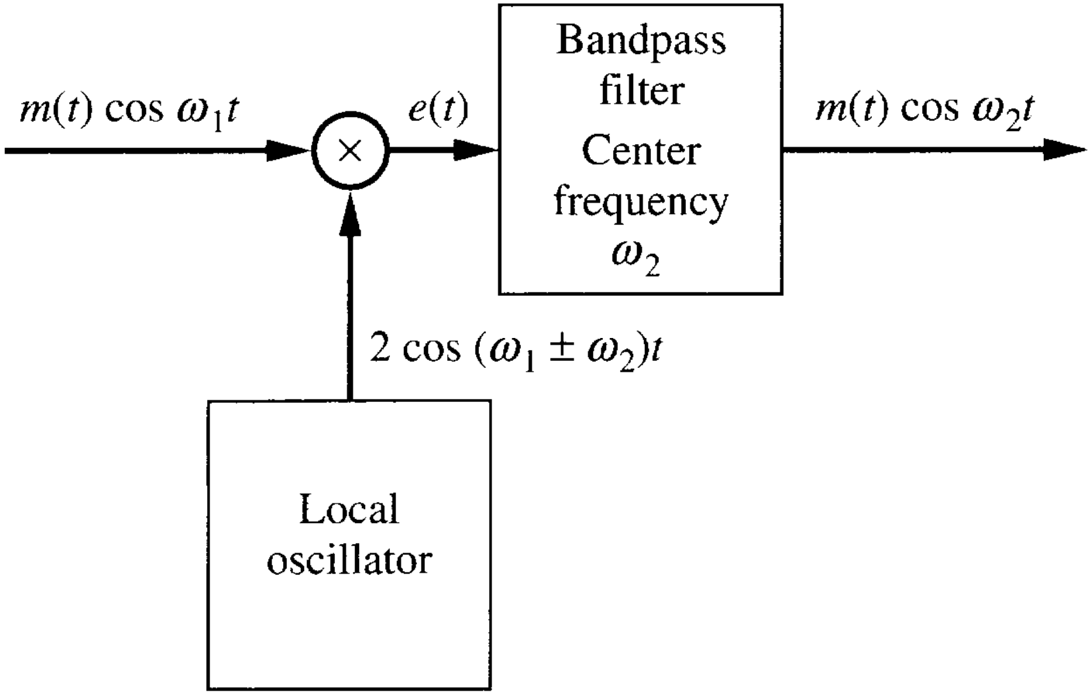
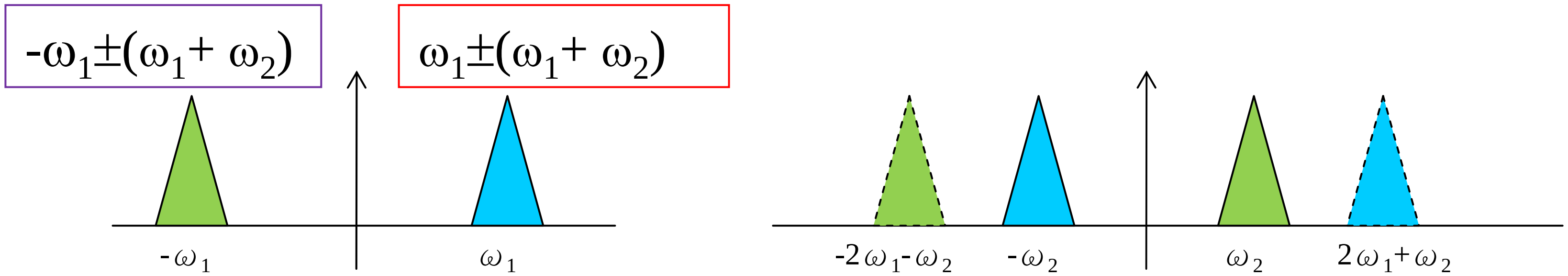


Figure 3.16
Mixer.

Mixing

- Mixing with $(\omega_1+\omega_2)$ [Original at ω_1] → ω_2



- *Problem?* Two input signals are translated to the same freq.! → **Image frequency**
- **Input 1** at $-\omega_1$ → $\pm(\omega_1+\omega_2)$ → ω_2 and $-(2\omega_1+\omega_2)$
- **Input 2** at $(\omega_1+2\omega_2)$ → $\pm(\omega_1+\omega_2)$ → ω_2 and $(2\omega_1+3\omega_2)$ ← Same ω_2 !

Image Frequency

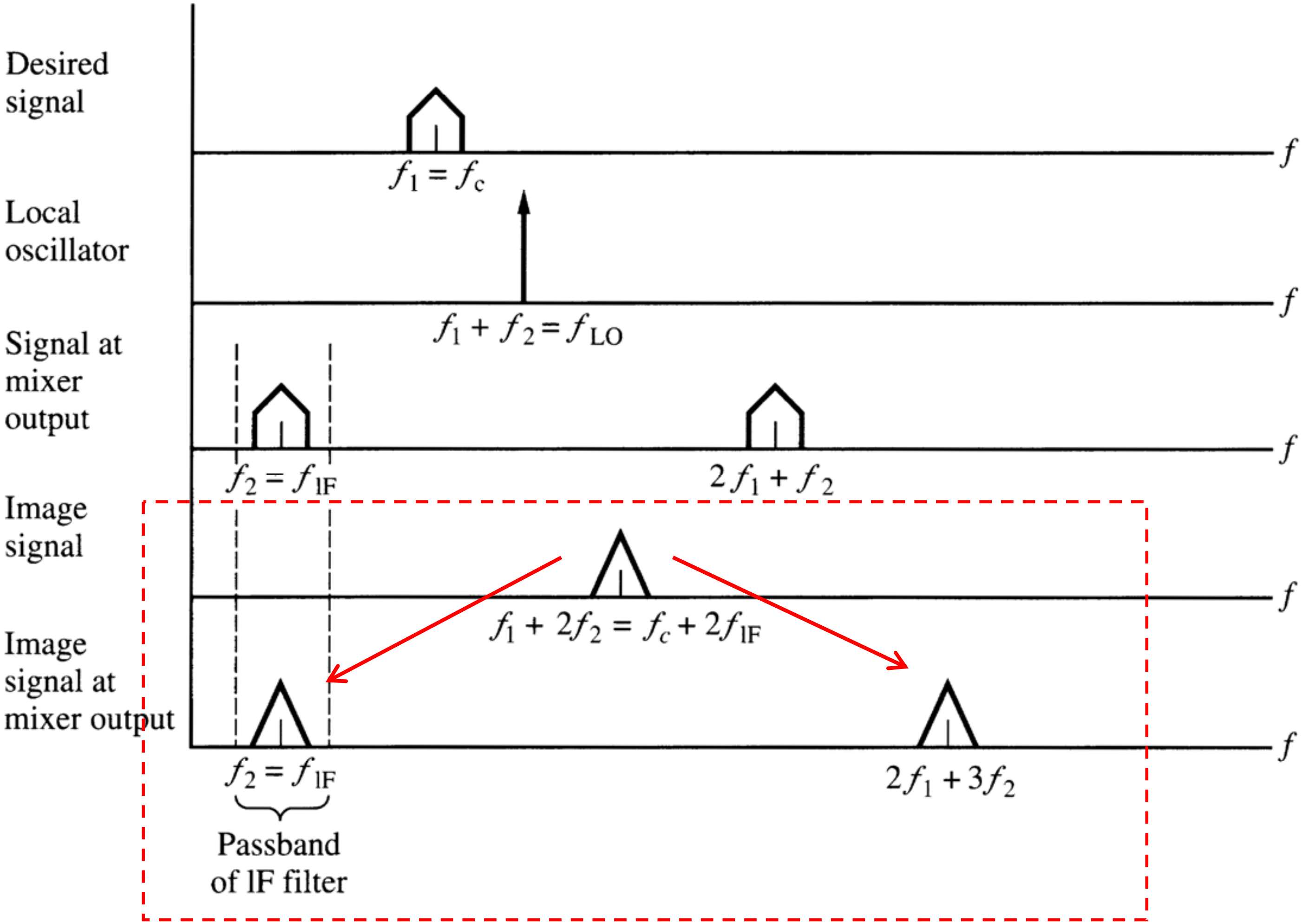


Figure 3.18
Illustration of image frequency (high-side tuning).

Superheterodyne Receiver

- A typical A broadcast receiver should perform the following functions in addition to "demodulation."

  1. Carrier-frequency tuning: select the desired signal (channel)

  2. Filtering: separate the desired signal from other modulated signals.

  3. Amplification: compensate for transmission loss. (Carlson, p.288)

- The super-heterodyne ("superhet") receiver fulfils the above requirements without using a high-gain tunable bandpass filter. (by E.H. Armstrong. 1918)
- *Remark*: It is difficult to construct a narrow bandpass filter at high frequency

Superheterodyne Receiver (2)

- It has two "amplification and filtering" sections prior to demodulation:

  (1) **RF**: radio freq; (2) **IF**: intermediate freq

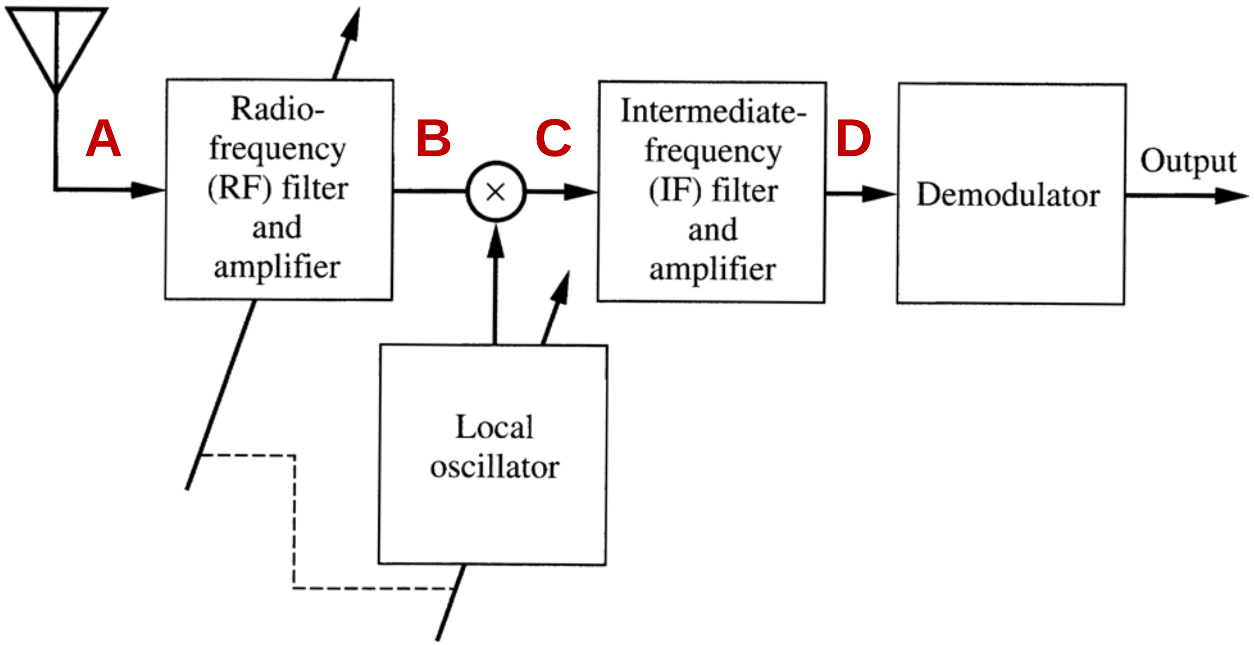


Figure 3.17
Superheterodyne receiver.

A. *At the antenna*:

The desired signal with carrier ω_c.

(undesired (image frequency) signals: $\omega_c+2\omega_{IF}$ or $\omega_c-2\omega_{IF}$)

B. *After the RF filter*:

Only the desired signal at ω_c can go through (a wide BPF filters out the image frequency with $2\omega_{IF}$). (Some near-by channels can go through but will be filtered out at IF)

C. *After the mixer*:

The desired signal at ω_{IF}. (There are other near-by channels)

D. *After the IF filter*:

Only the desired signal at ω_{IF} can go through. (a narrow BPF). (Other channels are filtered out)

- *Remarks*: **IF** freq is almost fixed (Carlson, p.289)
  - AM: IF~455 KHz; IFBW~10K; RF:0.54-1.6 MHz
  - FM: IF~10.7 MHz; IFBW~200K; RF:88-107.9 MHz

High-Side Tuning

- Two possible LO freqs. Choose the one with smaller LO tuning range.

  AM range: 540kHz ~ 1600kHz; IF: 455kHz

  Low-side tuning: $\omega_{LO} = \omega_c - \omega_{IF}$

  Range: 85kHz (540kHz – 455kHz) ~ 1145kHz (1600kHz – 455kHz)

  1 : 13.47

  High-side tuning: $\omega_{LO} = \omega_c + \omega_{IF}$

  Range: 995kHz (540kHz + 455kHz) ~ 2055kHz (1600kHz + 455kHz)

  1 : 2.07

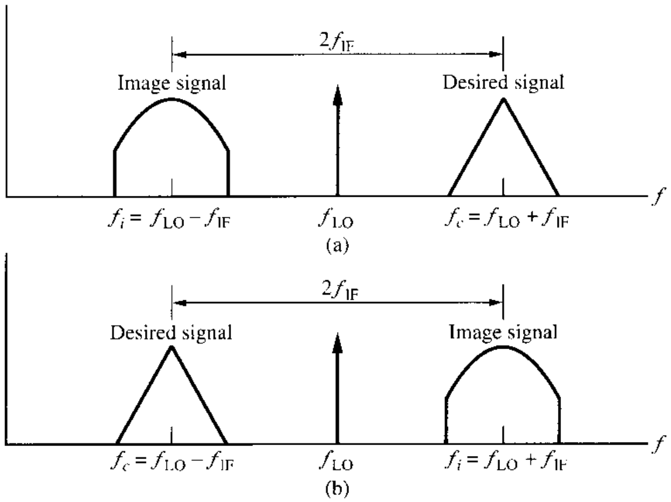


Figure 3.19 Relationship between f_c and f_i for (a) low-side tuning and (b) high-side tuning.