Principles of Communications
Lecture 12: Noise in Modulation Systems

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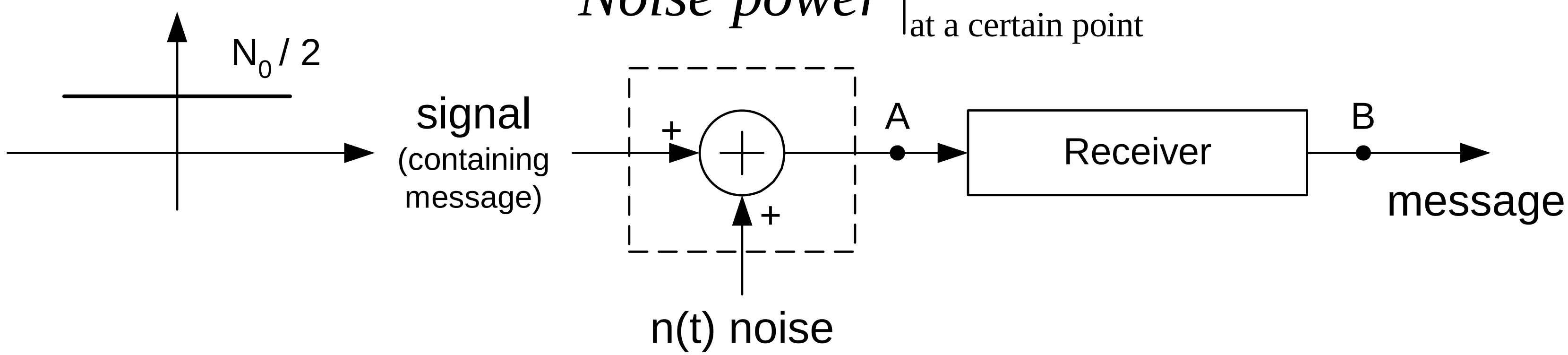
Outlines

- Signal-to-Noise Ratio
- Noise and Phase Errors in Coherent Systems
- Noise in Angle Modulation
- Threshold Phenomenon
- Noise in Pulse Code Modulation

Signal-to-Noise Ratio

- SNR is one of the most important parameters in communications. (The other is bandwidth).

$$SNR = \left. \frac{Signal\ power}{Noise\ power} \right|_{\text{at a certain point}}$$



- Channel Model: Additive, White, Gaussian Noise (AWGN). Independent of signal.
- Compare different receivers (modulations):
  Same SNR at A, compare SNR at B.

Noise in Baseband System

- The power spectral density is $\frac{N_0}{2}$. (Some books use N_0)
- **Physical meaning**: N_0 is the average noise power per unit bandwidth (single-sided psd) at the front end of the receiver.

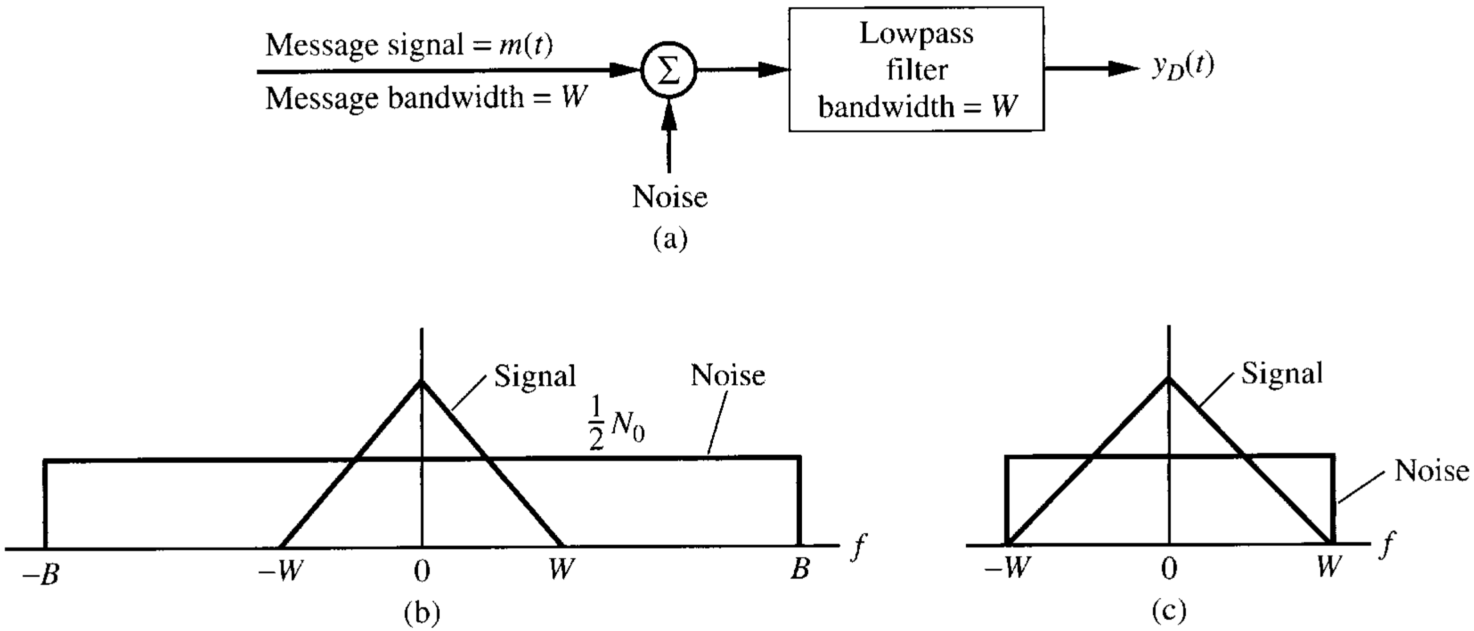


Figure 7.1
Baseband system. (a) Diagram. (b) Spectra at filter input. (c) Spectra at filter output.

- <u>Baseband model</u>: a basis for comparison (benchmark)

• Signal power = P_T watts. (transmitted power, modulated signal)

• Filter input noise power $=\int_{-B}^{B}\frac{1}{2}N_0 df = N_0 B$, and $(SNR)_i = \frac{P_T}{N_0 B}$.

• Filter output noise power$=\int_{-W}^{W}\frac{1}{2}N_0 df = N_0 W$, $(SNR)_o = \frac{P_T}{N_0 W}$.

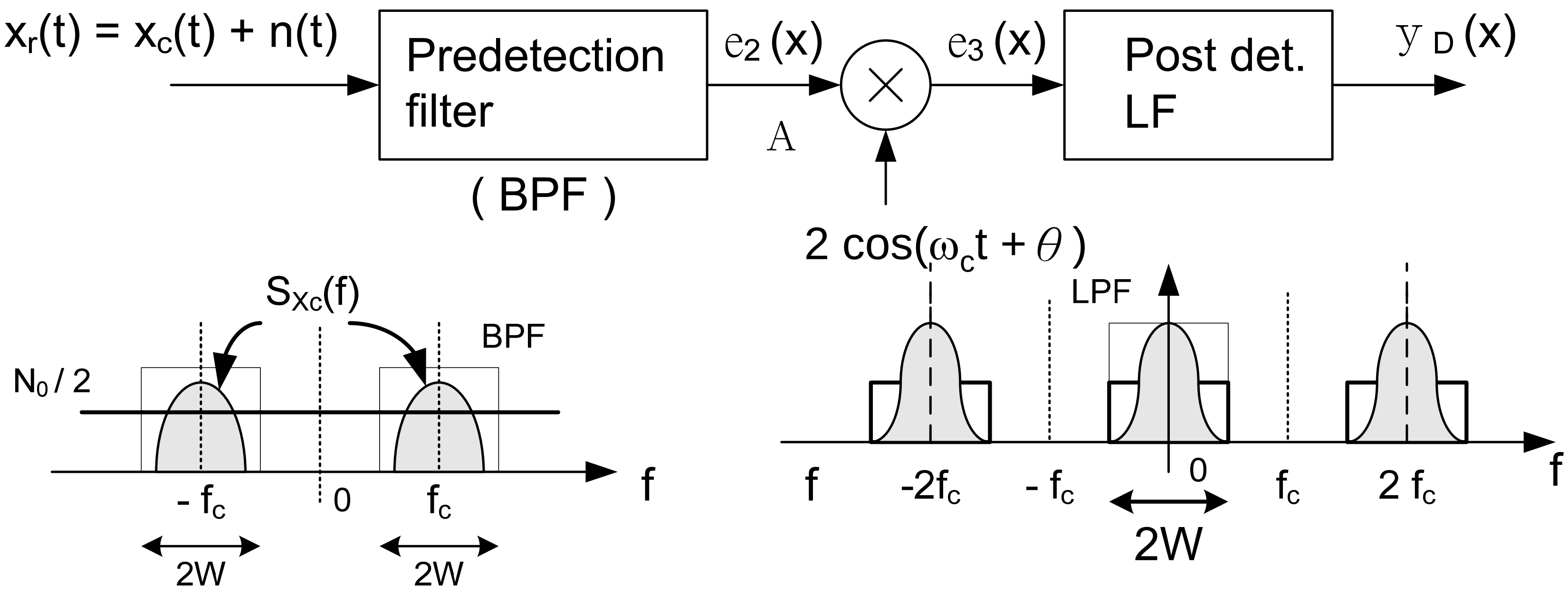
SNR enhancement $\frac{(SNR)_o}{(SNR)_i} = \frac{B}{W}$. (The filter reduces part of noise.)

- This is a basic operation in many comm systems – We filter out the *out-of-band* noise.

  This filtering does not change signal (power).

A. DSB-SC System

- Assume **coherent** detection.



$$x_r(t) = A_c m(t)\cos(\omega_c t + \theta) + n(t)$$

Bandpass filtering

Bandpass noise

$$e_2(t) = A_c m(t)\cos(\omega_c t + \theta) + n_c(t)\cos(\omega_c t + \theta) - n_s(t)\sin(\omega_c t + \theta)$$

$$\begin{cases} \text{Noise power: } \overline{n_0^2(t)} = \frac{1}{2}\overline{n_c^2(t)} + \frac{1}{2}\overline{n_s^2(t)} = 2N_0W. \\ \text{Transmitted signal power: } \frac{A_c^2}{2}\overline{m^2(t)}. \end{cases}$$

Passband

$$\Rightarrow \text{ Pre-detection SNR: } (SNR)_T = \frac{A_c^2\overline{m^2}}{4WN_0}. \text{ (For DSB-SC)}$$

$$e_3(t) = e_2(t)\cdot 2\cos(\omega_c t + \theta)$$

$$= A_c m(t)\{1+\cos(2(\omega_c t + \theta))\}$$

$$+ n_c(t)\{1+\cos(2(\omega_c t + \theta))\} - n_s(t)\sin(2(\omega_c t + \theta)).$$

$$\text{Through LPF } \Rightarrow y_0(t) = A_c m(t) + n_c(t).$$

$$\begin{cases} \text{Noise power: } \overline{n_c^2(t)} = 2N_0W. \text{ (Noise power remains the same.)} \\ \text{Signal power: } A_c^2\overline{m^2}. \text{ (Signal power "doubled")} \end{cases}$$

$$\Rightarrow \text{ Post-detection SNR: } \boxed{(SNR)_D = \frac{A_c^2\overline{m^2}}{2N_0W}}. \text{ (For DSB-SC)}$$

$$\text{Detection gain} \overset{\Delta}{=} \frac{(SNR)_D}{(SNR)_T} = 2 \quad (= 3dB).$$

- We assume coherent detection, i.e., the LO signal has the same freq and phase as the carrier.
- Does this detection (demodulation) provide a 3dB gain？ **Not quite.** Compare the equivalent baseband system.

- What is the *equivalent* baseband system?

$$\gamma = \frac{P_T}{N_0 W} \leftarrow \text{transmitted power} = \frac{1}{2} A_c^2 \overline{m^2}$$

$$= \frac{A_c^2 \overline{m^2}}{2 N_0 W} = (SNR)_D$$

- Why? The baseband noise is $N_0 W$ (BW=W).

  The passband noise is $2N_0 W$ (BW=$2W$).

  The detection gain cancels the noise *BW* increase.

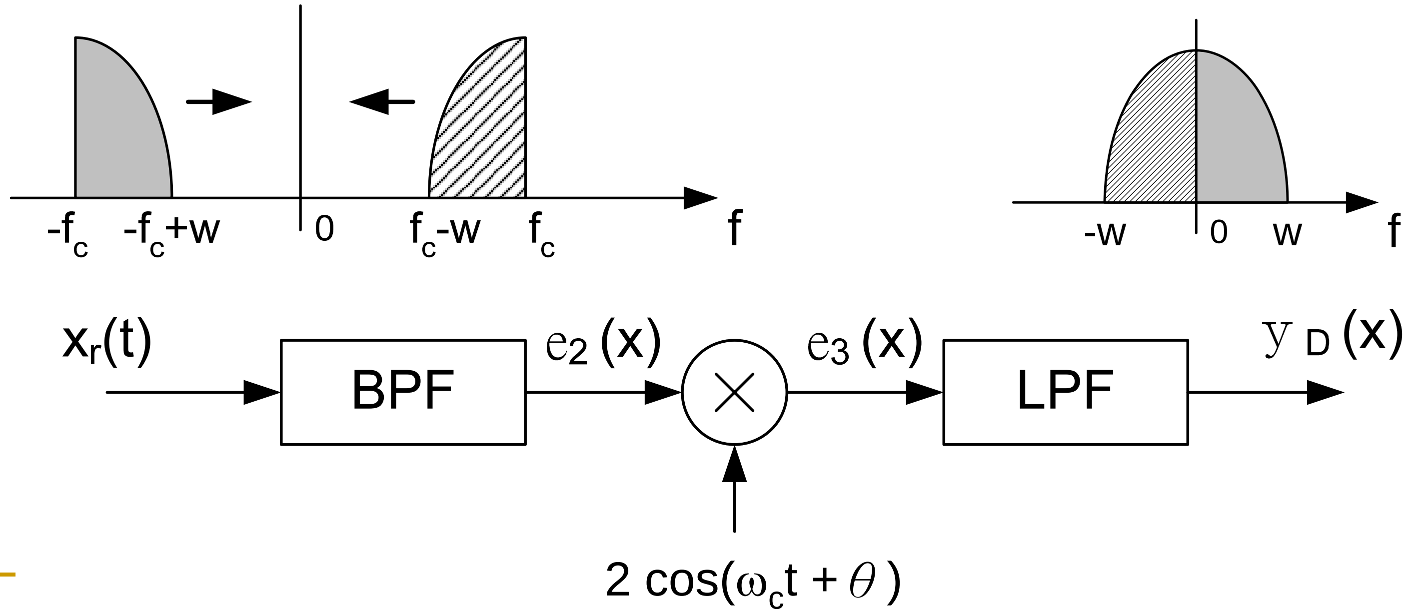
B. SSB System

- Assume coherent detection

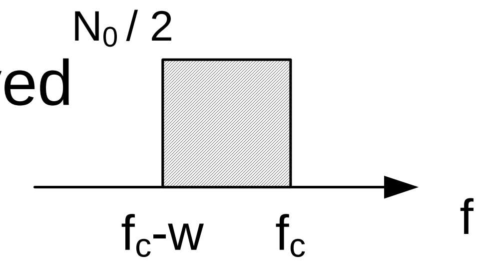
• The received signal (+ for the upper; − for the lower sideband):

$$x_r(t) = A_c[m(t)\cos(\omega_c t+\theta) \pm \hat{m}(t)\sin(\omega_c + \theta)] + n(t)$$

where $\hat{m}$: the Hilbert transform of $m(t)$.



Key: SSB BW = W not $2W$. Hence, the received noise power is reduced.



- Noise component: (Decompose into I and Q components)

$$n(t) = n_c(t)\cos(\omega_c t + \theta) - n_s(t)\sin(\omega_c t + \theta)$$

$$\text{Let } N_T \equiv \overline{n_0^2} = \overline{n_c^2} = \overline{n_s^2} = N_0 W$$

↑ BP noise

$$\text{Thus, } e_2(t) = [A_c m(t) + n_c(t)]\cos(\omega_c t + \theta) \pm [A_c \hat{m}(t) \mp n_s(t)]\sin(\omega_c t + \theta)$$

↓ ↑ don't care

$\times 2\cos(\omega_c t + \theta)$ coherent

e_3 ↓

$$\text{LPF} \rightarrow y_D(t) = A_c m(t) + \underline{n_c(t)} \leftarrow \text{noise term.}$$

- Assume that $m(t)$ is independent of n_c(t).

$$\begin{cases} \text{Post-detection signal power: } S_D = A_c^2 \overline{(m^2)}. \\ \text{Post-detection noise power: } N_D = \overline{(n_c^2)} = N_0 W. \end{cases}$$

$\Rightarrow$ Post-detection SNR: $\boxed{SNR_D = \dfrac{A_c^2 \overline{m^2}}{N_0 W}.}$ (For SSB-SC)

- What is the pre-detection SNR?

The transmitted signal power

$$S_T = \overline{\{A_c[m(t)\cos(\omega_c t + \theta) - \hat{m}(t)\sin(\omega_c t + \theta)]\}^2}$$

$$= A_c^2 \{\overline{[m(t)\cos(\omega_c t + \theta)]^2} + \overline{[\hat{m}(t)\sin(\omega_c t + \theta)]^2}$$

$$- 2\overline{m(t)\hat{m}(t)\cos(\omega_c t + \theta)\sin(\omega_c t + \theta)}$$

($m(t)$ and $\hat{m}(t)$ are orthogonal.)

($\cos(\omega_c t)$ and $\sin(\omega_c t)$ are orthogonal.)

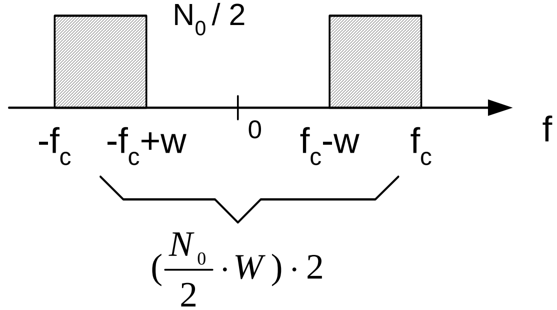
$$S_T = A_c^2\{\overline{m^2(t)}\underline{\langle\cos^2(\omega_c t+\theta)\rangle} + \overline{\hat{m}^2(t)}\underline{\langle\sin^2(\omega_c t+\theta)\rangle}$$

$$\downarrow \qquad\qquad \downarrow$$

$$\langle\frac{1}{2}+\frac{1}{2}\cos(2\omega_c t+2\theta)\rangle \qquad \langle\frac{1}{2}-\frac{1}{2}\cos(2\omega_c t+\theta)\rangle$$

$$= A_c^2\{\frac{1}{2}\overline{m^2(t)}+\frac{1}{2}\overline{\hat{m}^2(t)}\} \quad \text{(Notice that } \overline{m^2(t)}=\overline{\hat{m}^2(t)}.\text{)}$$

$$= A_c^2\overline{(m^2(t))} = S_D!$$



$\therefore$ The detection gain is

$$\frac{(SNR)_D}{(SNR)_T} = \frac{S_D / N_D}{S_T / N_T(=N_0W)} = 1. \text{ That is, } SNR_D = SNR_T.$$

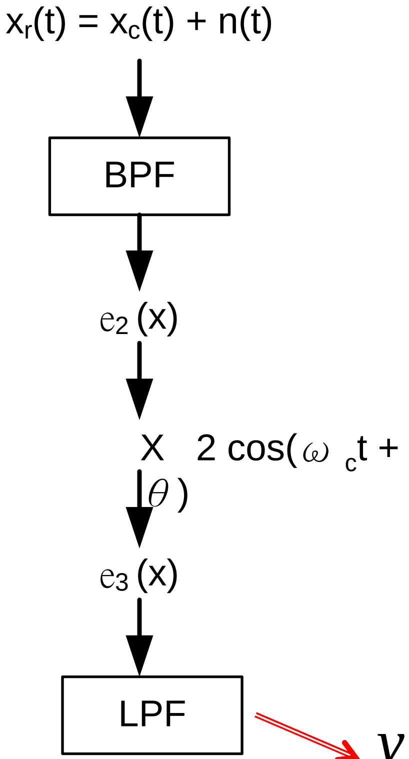
No gain! No loss ($\because N_T = N_D$)! (In DSB, $N_T = 2N_D$)

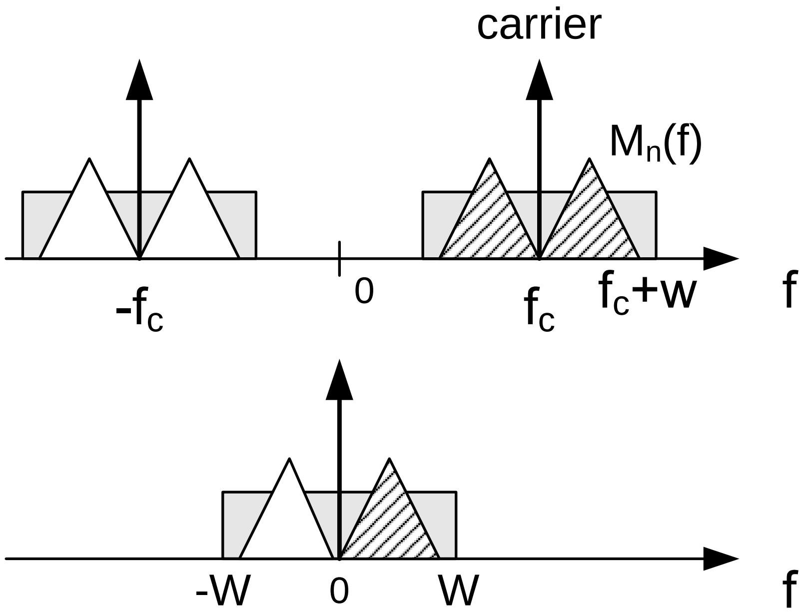
C. AM System -- Coherent

- Assume coherent detection
- Received signal

$$x_c(t) = A_c[1 + am_n(t)]\cos(\omega_c t + \theta).\quad a: \text{modulation index}$$

$$m_n: \text{normalized message}$$





$$y_D(t) = A_c am_n(t) + n_c(t) + A_c.\quad (A_c \text{ is removed.})$$

A_c is removed.

$\because$ 1) assume $\overline{m(t)} = 0$, we can thus remove dc term;

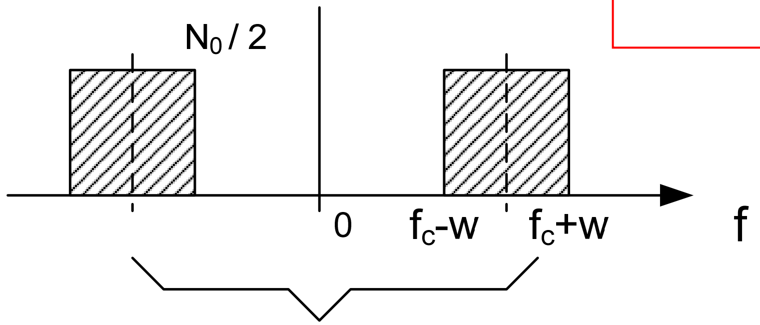
2) In reality, we cannot recover dc term of $m(t)$ anyway.

Thus we simply remove it.

- Post-detection signal power: $S_D = \overline{(A_c a m_n(t))^2} = A_c^2 a^2 \cdot \overline{m_n^2}$.

  Post-detection noise power: $N_D = \overline{n_c^2} = 2N_0 W$.

$\Rightarrow$ Post-detection SNR: $(SNR)_D = \dfrac{A_c^2 a^2 \overline{m_n^2}}{2N_0 W}$. (For AM)



$(\frac{N_0}{2} \cdot 2W) \cdot 2 = 2N_0 W$

- Now, let's calculate the pre-detection SNR. The Tx signal power:

$$S_T = \overline{\{A_c[1+am_n(t)]\cos(\omega_c t+\theta)\}^2}$$

$$= \overline{A_c^2\cdot\cos^2(\omega_c t+\theta)} + \overline{A_c^2 a^2 m_n^2(t)\cdot\cos^2(\omega_c t+\theta)}.$$

$$(\overline{m_n^2(t)} = m_n^2\ ,\quad \cos^2(\omega_c t+\theta) = \frac{1}{2} + \cos(2\omega_c t+\theta).)$$

$$P_T = S_T = \frac{1}{2}A_c^2 + \frac{1}{2}A_c^2 a^2 \overline{m_n^2}.$$

Bandpass noise power $N_T = 2N_0W$. $\Rightarrow (SNR)_T = \dfrac{1}{2}\cdot\dfrac{A_c^2 + A_c^2 a^2\overline{m_n^2}}{2N_0W}.$

$$\therefore \text{ The detection gain } = \frac{(SNR)_D}{(SNR)_T} = \frac{A_c^2 a^2\overline{m_n^2}}{\frac{1}{2}(A_c^2 + A_c^2 a^2\overline{m_n^2})} = \frac{2a^2\overline{m_n^2}}{1+a^2\overline{m_n^2}} < 1.$$

Recall, (power) efficiency $E_{ff} = \dfrac{a^2\overline{m_n^2}}{1+a^2\overline{m_n^2}}. \Rightarrow \dfrac{(SNR)_D}{(SNR)_T} = 2E_{ff}.$

- Ex: If $\overline{m_n^2} = 0.1$ and $a = 0.5$. $E_{ff} = \dfrac{(0.5)^2\cdot 0.1}{1+(0.5)^2\cdot 0.1} = 0.0244.$

$\Rightarrow$ Detection gain $=$ 0.0488! It is quite low!

D. AM System: Envelope Detection

- Envelope detector – incoherent
  - The received signal

$$e_2(t) = x_c(t) + n_0(t) \qquad x_c(t): \text{modulated signal}$$

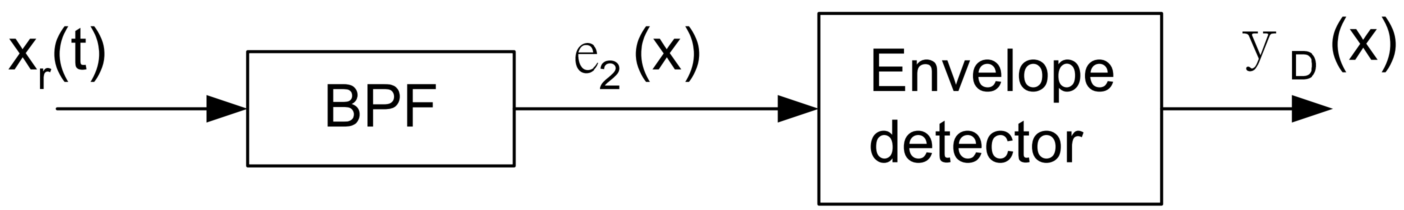
$$n_0(t): \text{narrow-band noise}$$

$$= A_c[1 + am_n(t)]\cos(\omega_c t + \theta) + \underline{n_c(t)\cos(\omega_c t + \theta) - n_s(t)\sin(\omega_c t + \theta)}.$$

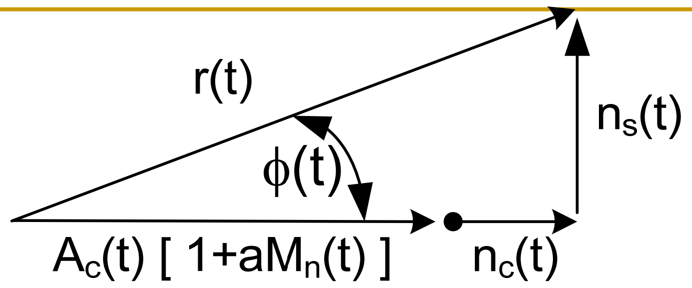
$$\text{Noise: } \overline{(n_c^2)} = \overline{(n_s^2)} = 2N_0 W.$$

$$\rightarrow e_2(t) = r(t)\cos[\omega_c t + \theta + \phi(t)].$$

$$\begin{cases} r(t) = \sqrt{\{A_c[1 + am_n(t)] + n_c(t)\}^2 + n_s^2(t)}. \\ \phi(t) = \tan^{-1}\left(\dfrac{n_s(t)}{A_c[1 + am_n(t)] + n_c(t)}\right). \end{cases}$$



Recall what we did for interference



- $y_D^{'}(t) = r(t)$ amplitude (envelope)

  $\downarrow$ remove dc (including the carrier)

$y_D(t) = r(t) - \bar{r}(t) \qquad \bar{r}(t)$: average

- Case 1: When $(SNR)_T$ is large (i.e., small noise)

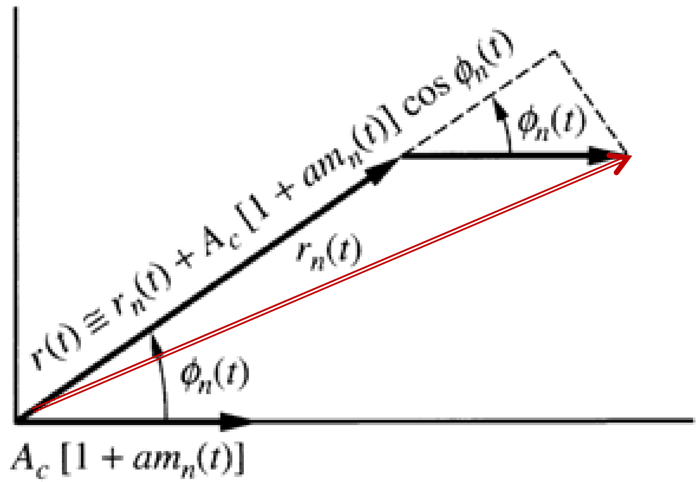
  $\left|A_c[1+am_n(t)]+n_c(t)\right| >> \left|n_s(t)\right|$

$\Rightarrow r(t) \cong A_c[1+am_n(t)]+n_c(t)$ most of the time

$\uparrow$ $\uparrow$ zero mean

$\Rightarrow y_D(t) \cong A_c a m_n(t) + n_c(t)$

$\uparrow$ same as coherent detection



- <u>Case 2</u>: When $(SNR)_T$ is small

The bandpass noise: $r_n(t)\cos(\omega_c t+\phi_n(t))$.

The enevelope detector input (θ is not important):

$$e_2(t)=A_c[1+am_n(t)]\cos(\omega_c t+\theta)+r_n(t)\cos(\omega_c t+\phi_n(t)+\theta)$$
$$=r(t)\cos(\omega_c t+\psi+\phi_n(t)).$$

Assume that $A_c[1+am_n(t)] << r_n(t)$ for most of time.

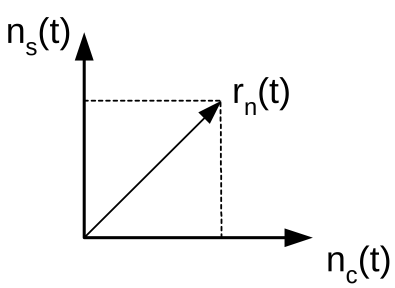
$$r(t)=\sqrt{[r_n(t)+A_c[1+am_n(t)]\cos\phi_n(t)]^2+[A_c(1+am_n(t)]\sin\phi_n(t)]^2}$$
$$\cong r_n(t)+A_c[1+am_n(t)]\cos\phi_n(t)$$
$$\downarrow \text{ remove "dc"}$$

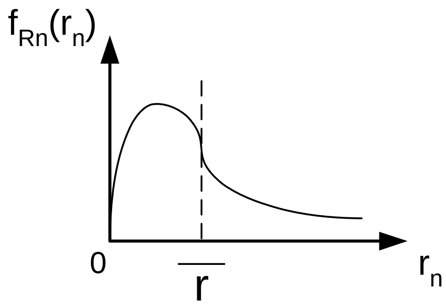
Random attenuation; the message is lost!

$$\underline{y_D(t)\cong r_n(t)+A_c[1+am_n(t)]\cos\phi_n(t)-\overline{r(t)}}$$

AM Noise Discussions

n(t) =





where $n_c(t)$ and $n_s(t)$ are all independent zero mean Gaussian

$$r_n(t) = \sqrt{n_c^2(t) + n_s^2(t)} = \text{Rayleigh-distr.}$$

$$\text{Now, } r(t) \sim r_n(t) + \underline{A_c[1 + am_n(t)]\cos\phi_n(t)} \qquad \cos\phi_n(t) : \text{random}$$

$$\text{message is lost!}$$

- What we try to show here is that when noise > (message) signal, "noise" becomes the <u>dominate</u> output component. In the output signal, no message signal is proportional to the message signal. (cf. Coherent detection:)

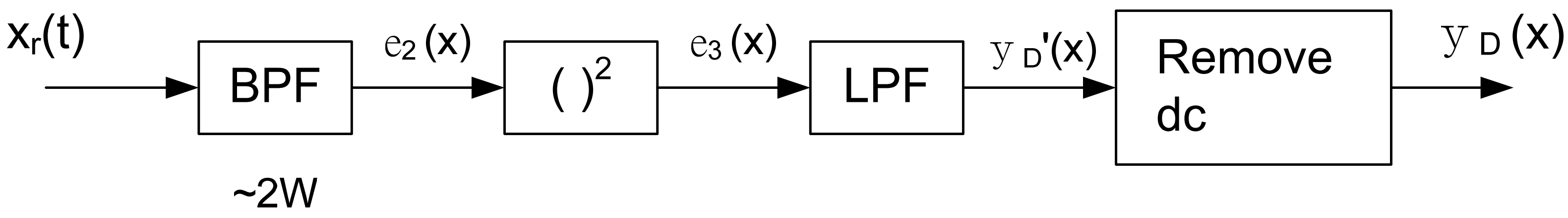
Threshold Effect

- This is the **threshold effect**. (Recall: The analysis in interference.) (∵The envelope detector is nonlinear, ∴the message is "lost" when SNR< threshold)
- *Remark*: It's difficult to calculate the exact $(\mathrm{SNR})_D$.

- Def. of threshold: A value of the carrier-to-noise ratio (or SNR) below which the noise performance of a detector deteriorates much more rapidly than proportionately to the carrier-to-noise ratio (or SNR). (Haykin&Moher, *Comm Systems*, p.215, 2010)

E. AM System: Square-Law Detection

- An example of simple nonlinear detector that we can calculate and thus the "threshold region" can be more precisely determined.



$$e_2(t) = x_c + n_0^{'}(t) \qquad n_0^{'}(t): \text{bandpass noise}$$

$$= A_c[1 + am_n(t)]\cos(\omega_c t) + n_c(t)\cos(\omega_c t) - n_s(t)\sin(\omega_c t).$$

$$e_3(t) = e_2^2(t)$$

$$e_3(t) = e_2^2(t) = \{A_c[1+am_n(t)] + n_c(t)\}^2 \cos^2(\omega_c t) -$$

$$2\{A_c[1+am_n(t)] + n_c(t)\} \cdot n_s(t)\cos(\omega_c t) \cdot \sin(\omega_c t) + n_s^2(t)\sin^2(\omega_c t)$$

$$= \{A_c[1+am_n(t)] + n_c(t)\}^2[\frac{1}{2} + \frac{1}{2}\cos(2\omega_c t)] -$$

$$\{A_c[1+am_n(t)] + n_c(t)\} \cdot n_s(t)\sin(2\omega_c t) + n_s^2(t)[\frac{1}{2} - \frac{1}{2}\cos(2\omega_c t)].$$

($\sin(2\omega_c t)$, $\cos(2\omega_c t)$: high freq. components)

↓ LPF

$$y_D'(t) \propto \{A_c[1+am_n(t)] + n_c(t)\}^2 + n_s^2(t)$$

$$= r^2(t) \qquad (= \text{envelope}^2)$$

$$= A_c^2 + 2A_c^2 a m_n(t) + A_c^2 a^2 m_n^2(t) + 2A_c n_c(t) + 2A_c a m_n(t) n_c(t)$$

$$+ n_c^2(t) + n_s^2(t).$$

$$y_D'(t) \xrightarrow{\text{remove dc}} y_D(t), \qquad \overline{m_n(t)} = 0, \quad \overline{n_c(t)} = \overline{n_s(t)} = 0$$

$$y_D(t) = \underline{2A_c^2 a m_n(t)} + A_c^2 a^2[m_n^2(t) - \overline{m_n^2(t)}] + 2A_c n_c(t) +$$

message

$$2A_c a m_n(t) n_c(t) + n_c^2(t) - \overline{n_c^2(t)} + n_s^2(t) - \overline{n_s^2(t)}.$$

Post-detection signal power: $S_D = 4A_c^4 a^2 \overline{(m_n^2)}$.

Post-detection noise power:

$$N_D = A_c^4 a^4 \overline{[(m_n^2(t) - m_n^2)^2]} + 4A_c^2 \overline{[1 + a m_n(t)]^2} \cdot \overline{n_c^2} +$$

$$\overline{(n_c^4)} - (\overline{n_c^2})^2 + \overline{(n_s^4)} - (\overline{n_s^2})^2.$$

$$\left[\overline{(x^2 - \overline{x^2})^2} = \overline{(x^4)} - 2\overline{x^2} \cdot \overline{x^2} + (\overline{x^2})^2 = \overline{(x^4)} - \overline{(x^2)}^2 \right]$$

(Assume the cross terms are either zero or can be neglected)

Note: $\overline{(n_c^4)} = 3 \cdot (\overline{n_c^2})^2$, if n_c is Gaussian. This can be shown using the moment generator.

$$N_D = A_c^4 a^4 \overline{[(m_n^2 - \overline{m_n^2})^2]} + 4A_c^2(1 + a^2 \overline{m_n^2}) \cdot \sigma_n^2 + 2 \cdot \sigma_n^4 + 2\sigma_n^4.$$

$$\Rightarrow (SNR)_D = \frac{S_D}{N_D} = \frac{4A_c^4 a^2 \overline{m_n^2}}{A_c^4 a^4 \overline{(m_n^2 - \overline{m_n^2})^2} + 4A_c^2(1 + a^2 m_n^2)\sigma_n^2 + 4\sigma_n^4}.$$

Example: message = sinusoidal

- Let $m_n(t) = \cos(\omega_m t)$. $\leftarrow$ not a random signal

$$\overline{(m_n^2(t) - \overline{m_n^2})^2} = \overline{[\cos^2(\omega_m t) - \frac{1}{2}]^2} = \overline{[\frac{1}{2}\cos(2\omega_m t)]^2} = \frac{1}{8}.$$

Assume this part in N_D can be neglected. ($\Leftarrow$ Will be discussed.)

Then, $N_D = 4A_c^2(1 + a^2 \frac{1}{2})\sigma_n^2 + 4\sigma_n^4.$

Note: If we examine $y_D^{'}(t)$ and $y_D(t)$

$$y_D^{'}(t) = \cancel{A_c^2} + 2A_c^2 a\cos(\omega_m t) + A_c^2 a^2[\cos^2(\omega_m t)]$$

$\downarrow$ remove dc $\quad +2A_c[1 + a\cos(\omega_m t)] \cdot n_c(t) + n_c^2(t) + n_s^2(t)$

$$(\cos^2\omega_m t = \cancel{\frac{1}{2}} + \frac{1}{2}\cos 2\omega_m t)$$

$$y_D(t) = \boxed{2A_c^2 a\cos(\omega_m t)} + \frac{1}{2}A_c^2 a^2 \cos(2\omega_m t)$$

message

$$+2A_c[1 + a\cos(\omega_m t)] \cdot n_c(t) + n_c^2(t) - \overline{n_c^2(t)} + n_s^2(t) - \overline{n_s^2(t)}$$

The term ( )$\cos(2\omega_m t)$ is called harmonic distortion.--not random noise.

Its power: $\underline{D_D = \frac{1}{8}A_c^4 a^4}(= \frac{1}{16}a^2 \cdot S_D)$

Example (conti.)

- Now, $N_D = 4A_c^2(1+a^2\frac{1}{2})\sigma_n^2 + 4\sigma_n^4$. And $S_D = 2A_c^4 a^2$.

$$\Rightarrow \underline{(SNR)_D} = \frac{2A_c^4 a^2}{2A_c^2(2+a^2)\sigma_n^2 + 4\sigma_n^4} = \frac{A_c^2 a^2 / \sigma_n^2}{\underline{(2+a^2) + 2(\sigma_n^2 / A_c^2)}}.$$

- Threshold effect illustration:

The total Tx power is $P_T = \overline{\{A_c[1+am_n(t)]\cdot\cos(w_c t)\}^2} = \frac{1}{2}A_c^2(1+a^2\overline{m_n^2}) = \frac{1}{2}A_c^2(1+\frac{1}{2}a^2)$.

$$(SNR)_D = 2(\frac{a}{2+a^2})^2 \frac{P_T / N_0 W}{1+(N_0 W / P_T)}. \qquad \text{(Baseband systems: } (SNR)_D = P_T / N_0 W\text{.)}$$

Case 1: If $\frac{P_T}{N_0 W} \square\ 1$, $(SNR)_D \cong 2(\frac{a}{2+a^2})^2 \cdot \frac{P_T}{N_0 W} \cong E\frac{P_T}{N_0 W}$ (□ Coherent)

Case 2: If $\frac{P_T}{N_0 W} \square\ 1$, $(SNR)_D \approx 2(\frac{a}{2+a^2})^2 \cdot (\frac{P_T}{N_0 W})^2$. (□ (□) $\cdot \frac{P_T}{N_0 W}$)

$$(\because 1+\frac{N_0 W}{P_T} \approx \frac{N_0 W}{P_T})$$

Example (conti.)

- For a 1st approximation, the performance of linear envelope detector ~ square-law detector
- For high *SNR* and *a*=1, the performance of envelope detector is better by ~1.8 dB.

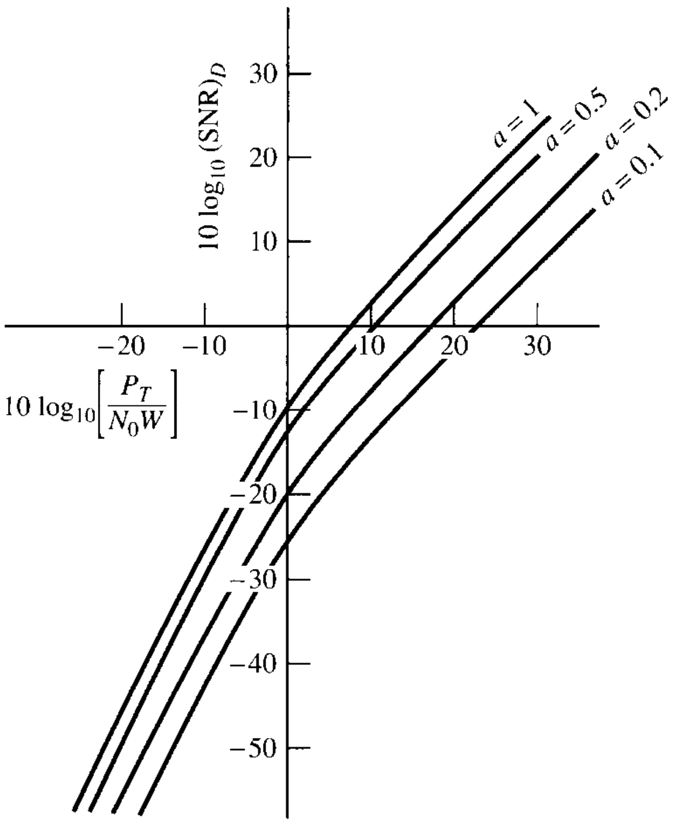


Figure 7.6
Performance of a square-law detector assuming sinusoidal modulation.

Discussions

- **General form**: (assume $\overline{(m_n^2 - \overline{m_n^2})^2} \cdot a^4$ is small)

$$(SNR)_D = \frac{4A_c^4 a^2 \overline{m_n^2}}{4A_c^2(1+a^2\overline{m_n^2})\sigma_n^2 + 4\sigma_n^4}$$

$$P_T = \frac{1}{2}A_c^2(1+a^2\overline{m_n^2}) \qquad (= \overline{\{A_c[1+am_n(t)]\cdot\cos(\omega_c t)\}^2})$$

$$E = \frac{a^2\overline{m_n^2}}{1+a^2\overline{m_n^2}}$$

$$(SNR)_D = \frac{4A_c^4 a^2\overline{m_n^2} \big/ (1+a^2\overline{m_n^2})}{4A_c^2(1+a^2\overline{m_n^2})\sigma_n^2 + 4\sigma_n^4 \big/ (1+a^2\overline{m_n^2})} = \frac{A_c^2 \cdot E}{\sigma_n^2 + \frac{1}{2}\sigma_n^4 \big/ P_T} = \frac{\frac{1}{2}A_c^2 \cdot E}{\frac{1}{2}\sigma_n^2 + \frac{1}{4}\sigma_n^4 \big/ P_T}$$

$$= \frac{\frac{1}{2}A_c^2 \cdot E}{P_T + N_0 W} \cdot \frac{P_T}{N_0 W} = \frac{a^2\overline{m_n^2}}{(1+a^2\overline{m_n^2})^2} \cdot \frac{P_T \big/ N_0 W}{1 + \frac{N_0 W}{P_T}}$$

Discussions (conti.)

- Special cases:

  If $P_T >> N_0 W,\ (SNR)_D \approx \dfrac{a^2 \overline{m_n^2}}{(1+a^2 \overline{m_n^2})^2} \cdot \dfrac{P_T}{N_0 W}$

  If $P_T << N_0 W,\ (SNR)_D \approx \dfrac{a^2 \overline{m_n^2}}{(1+a^2 \overline{m_n^2})^2} \cdot (\dfrac{P_T}{N_0 W})^2$

- *Example*: sinusoidal message → $\overline{m_n^2} = \dfrac{1}{2}$

  (previous example)

Summary

Table 7.1 Noise Performance Characteristics

System	Postdetection SNR	Transmission bandwidth
Baseband	$\frac{P_T}{N_0 W}$	W
DSB with coherent demodulation	$\frac{P_T}{N_0 W}$	$2W$
SSB with coherent demodulation	$\frac{P_T}{N_0 W}$	W
AM with envelope detection (above threshold) or AM with coherent demodulation. *Note*: E is efficiency	$\frac{EP_T}{N_0 W}$	$2W$
AM with square-law detection	$2\left(\frac{a^2}{2+a^2}\right)^2 \frac{P_T/N_0 W}{1+(N_0 W/P_T)}$	$2W$
PM above threshold	$k_p^2 \overline{m_n^2} \frac{P_T}{N_0 W}$	$2(D+1)W$
FM above threshold (without preemphasis)	$3D^2 \overline{m_n^2} \frac{P_T}{N_0 W}$	$2(D+1)W$
FM above threshold (with preemphasis)	$\left(\frac{f_d}{f_3}\right)^2 \overline{m_n^2} \frac{P_T}{N_0 W}$	$2(D+1)W$