HW4

Problem 3.51
The message signal is

$$m(t) = 3\sin 2\pi(10)t + 4\sin 2\pi(20)t$$

The derivative of the message signal is

$$\frac{dm(t)}{dt} = 60\pi\cos 2\pi(10)t + 160\pi\cos 2\pi(20t)$$

The maximum value of $dm(t)/dt$ is obviously 220π and the maximum occurs at $t = 0$. Thus

$$\frac{\delta_0}{T_s} \geq 220\pi$$

or

$$f_s \geq \frac{220\pi}{\delta_0} = \frac{220\pi}{0.05\pi} = 4400$$

Thus, the minimum sampling frequency is 4400 Hz.

Problem 3.52
Let A be the peak-to-peak value of the data signal. The peak error is 0.25% and the peak-to-peak error is $0.005A$. The required number of quantizating levels is

$$\frac{A}{0.005A} = 200 \leq 2^n = q$$

so we choose $q = 256$ and $n = 8$. The bandwidth is

$$B = 2Wk\log_2 q = 2Wk(8)$$

The value of k is estimated by assuming that the speech is sampled at the Nyquist rate. Then the sampling frequency is $f_s = 2W = 8$ kHz. Each sample is encoded into $n = 8$ pulses. Let each pulse be τ with corresponding bandwidth $\frac{1}{\tau}$. For our case

$$\tau = \frac{1}{nf_s} = \frac{1}{2Wn}$$

Thus the bandwidth is

$$\frac{1}{\tau} = 2Wn = 2W\log_2 q = 2nW = 2knW$$

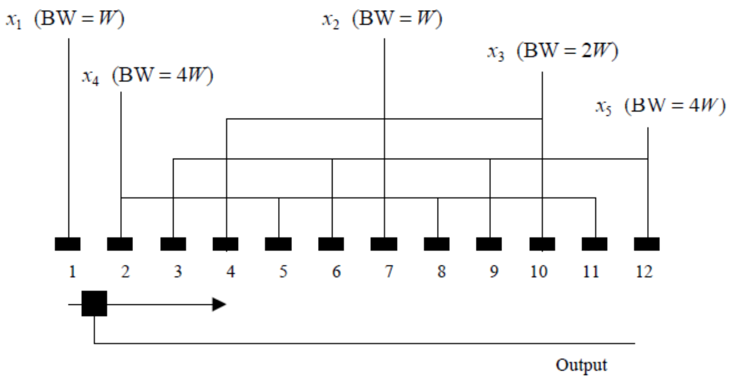


Figure 3.10: Commutator configuration for Problem 3.53.

and so $k = 1$. For $k = 1$

$$B = 2(8,000)(8) = 112 \text{ kHz}$$

Problem 3.54
The single-sided spectrum for $x(t)$ is shown in Figure 3.11.
From the definition of $y(t)$ we have

$$Y(s) = a_1 X(f) + a_2 X(f) * X(f)$$

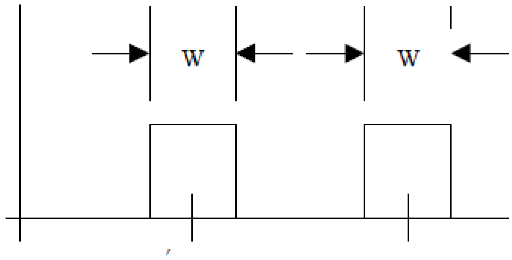


Figure 3.11: Single-sided spectrum for Problem 3.54.

The spectrum for $Y(f)$ is given in Figure 3.12. Demodulation can be a problem since it may be difficult to filter the desired signals from the harmonic and intermodulation distortion caused by the nonlinearity. As more signals are included in $x(t)$, the problem becomes more difficult. The difficulty with harmonically related carriers is that portions of the spectrum of $Y(f)$ are sure to overlap. For example, assume that $f_2 = 2f_1$. For this case, the harmonic distortion arising from the spectrum centered about f_1 falls exactly on top of the spectrum centered about f_2.

Problem 4.1

a. Split phase or bipolar RZ;

b. NRZ change or NRZ mark;

c. Split phase;

d. NRZ mark;

e. Polar RZ;

f. Unipolar RZ.

Problem 4.2
This is a matter of adapting Figures 4.2 top and bottom to the data sequence given in the problem statement.

Problem 4.8

Because $P_{RC}(f)$ is even, it follows that

$$\begin{aligned}
p_{RC}(t) &= F^{-1}[P_{RC}(f)] = 2\int_0^\infty P_{RC}(f)\cos(2\pi ft)\,df \\
&= 2T\int_0^{(1-\beta)/2T}\cos(2\pi ft)\,df + T\int_{(1-\beta)/2T}^{(1+\beta)/2T}\left\{1+\cos\left[\frac{\pi T}{\beta}\left(f-\frac{1-\beta}{2T}\right)\right]\right\}\cos(2\pi ft)\,df \\
&= 2T\frac{\sin\left(\frac{1-\beta}{T}\pi t\right)}{2\pi t} + T\frac{\sin\left(\frac{1+\beta}{T}\pi t\right)}{2\pi t} - T\frac{\sin\left(\frac{1-\beta}{T}\pi t\right)}{2\pi t} \\
&\quad + T\int_{(1-\beta)/2T}^{(1+\beta)/2T}\cos\left[\frac{\pi T}{\beta}\left(f-\frac{1-\beta}{2T}\right)\right]\cos(2\pi ft)\,df \\
&= T\frac{\sin\left(\frac{1-\beta}{T}\pi t\right)}{2\pi t} + T\frac{\sin\left(\frac{1+\beta}{T}\pi t\right)}{2\pi t} \\
&\quad + \frac{T}{2}\int_{(1-\beta)/2T}^{(1+\beta)/2T}\left\{\cos\left[\frac{\pi T}{\beta}\left(f-\frac{1-\beta}{2T}\right)+2\pi ft\right]+\cos\left[\frac{\pi T}{\beta}\left(f-\frac{1-\beta}{2T}\right)-2\pi ft\right]\right\}df \\
&= T\frac{\sin\left(\frac{1-\beta}{T}\pi t\right)}{2\pi t} + T\frac{\sin\left(\frac{1+\beta}{T}\pi t\right)}{2\pi t} \\
&\quad + \frac{T}{2}\left.\frac{\sin\left[\frac{\pi T}{\beta}\left(f-\frac{1-\beta}{2T}\right)+2\pi ft\right]}{\frac{\pi T}{\beta}+2\pi t}\right|_{(1-\beta)/2T}^{(1+\beta)/2T} + \frac{T}{2}\left.\frac{\sin\left[\frac{\pi T}{\beta}\left(f-\frac{1-\beta}{2T}\right)-2\pi ft\right]}{\frac{\pi T}{\beta}-2\pi t}\right|_{(1-\beta)/2T}^{(1+\beta)/2T} \\
&= T\frac{\sin\left(\frac{1-\beta}{T}\pi t\right)}{2\pi t} + T\frac{\sin\left(\frac{1+\beta}{T}\pi t\right)}{2\pi t} \\
&\quad + \frac{T}{2}\frac{\sin\left[\frac{\pi T}{\beta}\left(\frac{1+\beta}{2T}-\frac{1-\beta}{2T}\right)+\pi\frac{1+\beta}{T}t\right]}{\pi T/\beta+2\pi t} - \frac{T}{2}\frac{\sin\left[\frac{\pi T}{\beta}\left(\frac{1-\beta}{2T}-\frac{1-\beta}{2T}\right)+\pi\frac{1-\beta}{T}t\right]}{\pi T/\beta+2\pi t} \\
&\quad + \frac{T}{2}\frac{\sin\left[\frac{\pi T}{\beta}\left(\frac{1+\beta}{2T}-\frac{1-\beta}{2T}\right)-\pi\frac{1+\beta}{T}t\right]}{\pi T/\beta-2\pi t} - \frac{T}{2}\frac{\sin\left[\frac{\pi T}{\beta}\left(\frac{1-\beta}{2T}-\frac{1-\beta}{2T}\right)-\pi\frac{1-\beta}{T}t\right]}{\pi T/\beta-2\pi t} \\
&= \frac{\sin(\pi t/T)\cos(\pi\beta t/T)}{\pi t/T} + \frac{1}{2\pi t/T}\left[\frac{1}{T/2\beta t-1}-\frac{1}{T/2\beta t+1}\right]\cos\left(\frac{\pi\beta t}{T}\right)\sin\left(\frac{\pi t}{T}\right) \\
&= \left[1-\frac{1}{1-(T/2\beta t)^2}\right]\cos(\pi\beta t/T)\frac{\sin(\pi t/T)}{\pi t/T} \\
&= \left[-\frac{(T/2\beta t)^2}{1-(T/2\beta t)^2}\right]\cos(\pi\beta t/T)\frac{\sin(\pi t/T)}{\pi t/T} = \frac{\cos(\pi\beta t/T)}{1-(2\beta t/T)^2}\mathrm{sinc}(t/T)
\end{aligned}$$

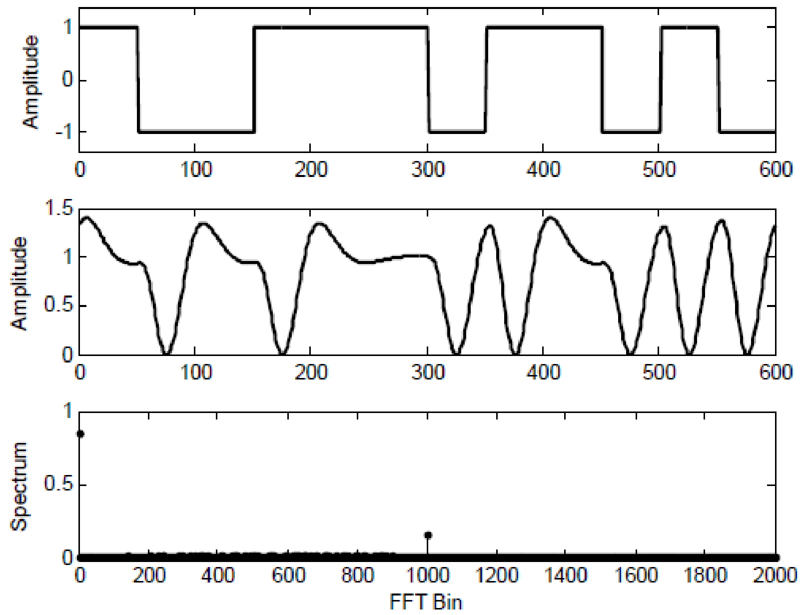
Problem 4.9

From (4.37) the flat-top portion of the spectrum goes to $f = 1/2T$ and the transition region is 0 in frequency extent. The remaining spectrum is 0 for $1/2T < f \leq 1/T$. Setting $\beta = 0$ in (4.36) clearly gives $p_{RC}(t) = \mathrm{sinc}(t/T)$.

Problem 4.19

The modified program follows with the spectrum shown in Fig. 4.2. The spectral line at the bit rate does not appear to be much different than that of Fig 4.16.

```
% Problem 4-19
%
nsym = 1000; nsamp = 50; lambda = 0.7;
[b,a] = butter(3,2*lambda/nsamp);
l = nsym*nsamp;                % Total sequence length
y = zeros(1,l-nsamp+1);        % Initalize output vector
x =2*round(rand(1,nsym))-1;    % Components of x = +1 or -1
```




```
for i = 1:nsym                    % Loop to generate info symbols
k = (i-1)*nsamp+1;
y(k) = x(i);
end
datavector1 = conv(y,ones(1,nsamp)); % Each symbol is nsamp long
subplot(3,1,1), plot(datavector1(1,200:799),'k', 'LineWidth', 1.5)
axis([0 600 -1.4 1.4]), ylabel('Amplitude')
filtout = filter(b,a,datavector1);
% datavector2 = filtout.*filtout;
datavector2 = abs(filtout);
subplot(3,1,2), plot(datavector2(1,200:799),'k', 'LineWidth', 1.5), ylabel('Amplitude')
y = fft(datavector2); yy = abs(y)/(nsym*nsamp);
subplot(3,1,3), stem(yy(1,1:2*nsym),'k.')
xlabel('FFT Bin'), ylabel('Spectrum')
% End of script file.
```