HW3

Problem 3.44

The phase plane is defined by

$$\psi = \Delta\omega - K_t \sin\psi(t)$$

at $\psi = 0$, $\psi = \psi_{ss}$, the steady-state phase error. Thus

$$\psi_{ss} = \sin^{-1}\left(\frac{\Delta\omega}{K_t}\right) = \sin^{-1}\left(\frac{\Delta\omega}{2\pi(100)}\right)$$

For $\Delta\omega = 2\pi(30)$

$$\psi_{ss} = \sin^{-1}\left(\frac{30}{100}\right) = 17.46 \text{ degrees}$$

For $\Delta\omega = 2\pi(50)$

$$\psi_{ss} = \sin^{-1}\left(\frac{50}{100}\right) = 30 \text{ degrees}$$

For $\Delta\omega = 2\pi(80)$

$$\psi_{ss} = \sin^{-1}\left(\frac{80}{100}\right) = 53.13 \text{ defrees}$$

For $\Delta\omega = -2\pi(80)$

$$\psi_{ss} = \sin^{-1}\left(\frac{-80}{100}\right) = -53.13 \text{ degrees}$$

For $\Delta\omega = 2\pi(120)$, there is no stable operating point and the frequency error and the phase error oscillate (PLL slips cycles continually).

Problem 3.48

From the definition of the transfer function

$$\frac{\Theta(s)}{\Phi(s)} = \frac{K_t F(s)}{s + K_t F(s)} = \frac{K_t\left(\frac{s+a}{s+\lambda a}\right)}{s + K_t\left(\frac{s+a}{s+\lambda a}\right)}$$

which is

$$\frac{\Theta(s)}{\Phi(s)} = \frac{K_t(s+a)}{s(s+\lambda a) + K_t(s+a)} = \frac{K_t(s+a)}{s^2 + (K_t + \lambda a)s + K_t a}$$

Therefore

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + (K_t + \lambda a)s + K_t a$$

This gives

$$\omega_n = \sqrt{K_t a}$$

and

$$\zeta = \frac{K_t + \lambda a}{2\sqrt{K_t a}}$$