Principles of Communications
Lecture 8: Baseband Communication Systems

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Outlines

- Introduction
- Line codes
- Effects of filtering
- Pulse shaping toward zero ISI
- Zero-forcing equalization
- Eye diagrams
- Synchronization
Introduction

- Digital vs. analog signals
- Analog $\rightarrow$ sampling $\rightarrow$ quantization $\rightarrow$ digital
- Baseband vs. passband
- Channel distortion – ISI

(Channel) bandwidth limitation

Figure 4.1
Block diagram of a baseband digital data transmission system.
Line Codes

- Baseband *data format* used to represent digital data (for transmission purpose).
- Examples are given on the next page
- **Operation**: time or frequency *shaping*
- **Purposes**: usually to cope with the channel limitations (or provide extra function such as synchronization)
- Needed for certain applications
- Non-return-to-zero (NRZ) change
- NRZ mark (data1 -> change in level; data0 -> no change)
- Unipolar return-to-zero (URZ) (<1/2 width pulses>)
- Polar RZ
- Bipolar RZ (“0” -> 0 level; “1” -> alternate sign)
- Split phase (Manchester) (“1” -> level A to level –A at 1/2 interval; “0” -> level –A to level A at 1/2 interval)

Figure 4.2
Abbreviated list of binary data formats.
Purposes of Line Codes

- Self synchronization
- Proper power spectrum
- Transmission bandwidth
- Transparency
- Error detection capability
- Good error probability performance
Power Spectra (I)

The transmitted signal is a pulse train:

\[ X(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT - \Delta). \]

The amplitudes can be viewed as random variables with

\[ R_m = \langle a_k a_{k+m} \rangle \quad m = 0, \pm 1, \pm 2, \ldots \]

The autocorrelation function of the waveform is

\[ R_X(\tau) = \sum_{m=-\infty}^{\infty} R_m r(\tau - mT), \]

in which \( r(\tau) = \frac{1}{T} \int_{-\infty}^{\infty} p(t + \tau) p(t) dt. \)
Power Spectra (II)

The power spectral density is the Fourier transform of $R_X(\tau)$:

\[
S_X(f) = \mathcal{F}\mathcal{T}[R_X(\tau)] = \mathcal{F}\mathcal{T}\left[ \sum_{m=-\infty}^{\infty} R_m r(\tau - mT) \right] \\
= \sum_{m=-\infty}^{\infty} R_m \mathcal{F}\mathcal{T}[r(\tau - mT)] = \sum_{m=-\infty}^{\infty} R_m S_r(f) e^{-j2\pi mf T} \\
= S_r(f) \sum_{m=-\infty}^{\infty} R_m e^{-j2\pi mf T}.
\]

Note that $S_r(f) = \mathcal{F}\mathcal{T}[r(\tau)] = \mathcal{F}\mathcal{T}\left[ \frac{1}{T} p(-t) * p(t) \right] = \frac{|P(f)|^2}{T}$.
Example 1: NRZ

- Assume the message $m[n]$ is random (white noise) with equally “0” and “1” values.
- Step 1: Compute $R_m$ based on the above assumption and “format”
- Step 2: Compute $r(t)$ based on the pulse shape.

1. NRZ, 50% "0" and 50% "1".

$$R_m = \frac{1}{2} A^2 + \frac{1}{2} (-A)^2 = A^2, m = 0;$$

$$R_m = \frac{1}{4} A^2 + \frac{1}{4} A(-A) + \frac{1}{4} (-A)A + \frac{1}{2} (-A)^2 = 0, m \neq 0.$$ 

$$p(t) = \Pi(t / T) \rightarrow P(f) = Tsinc(Tf)$$

Therefore, $S_{NRZ}(f) = A^2 S_r(f) = A^2 Tsinc^2(Tf).$
Example 2: Unipolar RZ

2. Unipolar RZ, 50% 1-level and 50% 0-level.

\[ R_m = \begin{cases} \frac{1}{2} A^2 + \frac{1}{2} (0)^2 = \frac{1}{2} A^2, & m = 0 \\ \frac{1}{4} A \cdot A + \frac{1}{4} A \cdot 0 + \frac{1}{4} 0 \cdot A + \frac{1}{4} 0 \cdot 0 = \frac{1}{4} A^2, & m \neq 0 \end{cases} \]

\[ p(t) = \Pi(2t / T) \rightarrow P(f) = \frac{T}{2} \text{sinc}(\frac{T}{2} f) \]

\[ S_{URZ}(f) = \frac{T}{4} \text{sinc}^2(\frac{T}{2} f) \left[ \frac{1}{2} A^2 + \frac{1}{4} A^2 \sum_{m=-\infty}^{m=\infty} e^{-j2\pi m T f} \right] \]

\[ = \frac{T}{4} \text{sinc}^2(\frac{T}{2} f) \left[ \frac{1}{4} A^2 + \frac{1}{4} A^2 \sum_{m=-\infty}^{m=\infty} e^{-j2\pi m T f} \right] \]

\[ = \frac{T}{4} \text{sinc}^2(\frac{T}{2} f) \left[ \frac{1}{4} A^2 + \frac{1}{4} A^2 \sum_{n=-\infty}^{n=\infty} \delta(f - \frac{n}{T}) \right] \cdot \sum_{m=-\infty}^{m=\infty} e^{-j2\pi m T f} = \frac{1}{T} \sum_{n=-\infty}^{n=\infty} \delta(f - \frac{n}{T}) \]
Figure 4.3
Power spectra for line-coded binary data formats.
Inter-symbol Interference (ISI)

- **Intersymbol interference** refers to one specific type of *distortion*. It is typically due to insufficient bandwidth of the channel. *Note:* Narrow BW $\rightarrow$ Long time-duration

- Example: Rectangular waveforms through a lowpass RC filter. The neighboring pulses smear out and interfere with each other.

![Graphs showing inter-symbol interference](image)

*Figure 4.4*  
Response of a lowpass RC filter to a positive rectangular pulse followed by a negative rectangular pulse to illustrate the concept of ISI. (a) $T/RC = 20$. (b) $T/RC = 2$.  

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Figure 4.5
Data sequences formatted with various line codes passed through a channel represented by a second-order lowpass Butterworth filter of bandwidth 1 bit rate.
Figure 4.6
Data sequences formatted with various line codes passed through a channel represented by a second-order lowpass Butterworth filter of bandwidth $\frac{1}{2}$ bit rate.
ISI Reduction

- **Pulse shaping**: Assume the channel is ideal (flat) with the minimum BW \( W \). What shape of “pulse” warrants ISI-free transmission?

- **Equalization**: If the channel is non-ideal (not ideally flat LPF), an equalizer helps in reducing ISI.
Pulse Shaping

Consider $2W$ independent samples per second are transmitted through a channel with bandwidth $W$ Hz. The output is as follows.

\[ y(t) = \sum_{n=-\infty}^{\infty} y_n(t) = \sum_{n=-\infty}^{\infty} a_n \text{sinc} \left[ 2W \left( t - \frac{n}{2W} \right) \right]. \]

The samples at $t_m = m / 2W$ are ISI-free, $\because \text{sinc}(m - n) = 0$.

Are there other pulse shapes warrant ISI-free?

![Pulse Shaping Diagram](image-url)
Raised Cosine Family

- **Raised Cosine**: A example of ISI-free pulses – raised cosine spectra at BW edges.

\[
P_{RC}(f) = \begin{cases} 
    T, & |f| \leq \frac{1-\beta}{2T} \\
    \frac{T}{2} \left\{ 1 + \cos \left[ \frac{\pi T}{\beta} \left( |f| - \frac{1-\beta}{2T} \right) \right] \right\} & \frac{1-\beta}{2T} < |f| \leq \frac{1+\beta}{2T} \\
    0 & |f| > \frac{1+\beta}{2T}
\end{cases}
\]

- **Roll-off factor** $\beta$: $\beta=0 \rightarrow$ rectangular pulse (sinc)  
  $\beta=1 \rightarrow$ “cosine” in freq; time pulse has narrow main lobe with very low sidelobes
Time pulse of the raised cosine:

\[ p_{RC}(t) = \frac{\cos(\pi \beta t / T)}{1 - (2\beta t / T)^2} \text{sinc}\left(\frac{t}{T}\right) \], \beta \text{ is called the roll-off factor.}
Nyquist’s Pulse Shaping Criterion

A pulse shape $p(t)$ with a Fourier transform $P(f)$ satisfying

$$\sum_{k=-\infty}^{\infty} P(f + \frac{k}{T}) = T, \quad |f| \leq \frac{1}{2T}$$

then its sampled values $p(nT) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$.

Figure 4.8
Illustration that (a) a triangular spectrum satisfies (b) Nyquist’s zero ISI criterion.
Proof of Nyquist's Pulse Shaping Criterion

\[ p(t) = \int_{-\infty}^{\infty} P(f) e^{j2\pi ft} \, df \]

Sampling at \( nT \), \( p(nT) = \int_{-\infty}^{\infty} P(f) e^{j2\pi fnT} \, df \)

\[ p(nT) = \sum_{k=-\infty}^{\infty} \int_{2k-1/2T}^{2k+1/2T} P(f) e^{j2\pi fnT} \, df \]

\[ = \sum_{k=-\infty}^{\infty} \int_{-1/2T}^{1/2T} P\left(u + \frac{k}{T}\right) e^{j2\pi f\left(\frac{u}{T}\right)} \, du; \quad u = f - \frac{k}{T} \]

\[ = \int_{-1/2T}^{1/2T} \sum_{k=-\infty}^{\infty} P\left(u + \frac{k}{T}\right) e^{j2\pi f\left(\frac{u}{T}\right)} \, du \]

\[ = \int_{-1/2T}^{1/2T} Te^{j2\pi f\left(\frac{u}{T}\right)} \, du = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \]
Consider the combined effect of three filters, we would like to have $v(t)$ to be ISI-free.

\[ x(t) = \sum_{k=-\infty}^{\infty} a_k h_T(t - kT). \]

\[ v(t) = y(t) * h_R(t) = [x(t) * h_C(t)] * h_R(t). \]

**Figure 4.9**
Transmitter, channel, and receiver cascade illustrating the implementation of a zero-ISI communication system.
Transmitter and Receiver Filters

Let \( v(t) = A \sum_{k=-\infty}^{\infty} a_k p_{RC}(t - kT - t_d) \)

where \( A \) represents a scale factor and \( t_d \) a possible delay.

\( A p_{RC}(t - t_d) = h_T(t) * h_C(t) * h_R(t) \), or in frequency domain,

\( AP_{RC}(f)e^{-j2\pi f t_d} = H_T(f)H_C(f)H_R(f) \).

If \( H_C(f) \) is given, then \( |H_T(f)||H_R(f)| = 1/|H_C(f)| \)

The overall spectrum should satisfy the Nyquist criterion to avoid ISI. \( \Rightarrow \) A filter design problem.

- Practically, \( H_C(f) \) is unknown of time-varying
Let $|H_T(f)| = |H_R(f)| = 1/|H_C(f)|^{1/2}$

Bit rate = 5000 bps; channel filter 3-dB frequency = 2000 Hz; no. of poles = 1

**Figure 4.10**
Transmitter and receiver filter amplitude responses that implement the zero-ISI condition assuming a first-order Butterworth channel filter and raised cosine pulse shapes.
Zero-Forcing Equalization

- Now, assume digital processing at the receiver.
- **Purpose:** Design an FIR filter that compensates for the channel distortion → zero-ISI.
- Show an example of zero-ISI equalizer design.

![Diagram](image)

**Figure 4.11**
A transversal filter implementation for equalization of ISI.
Let the combined impulse response after the channel be $p_c(t)$, Pass such a pulse through our finite-length equalizer, then

$$p_{eq}(t) = \sum_{n=-N}^{n} \alpha_n p_c(t - n\Delta).$$

If the resulting pulse satisfies the zero-ISI condition, ISI-free transmission is achieved. Assume that $\Delta = T$, the condition is

$$p_{eq}(mT) = \sum_{n=-N}^{n} \alpha_n p_c[(m-n)T] = \begin{cases} 1, & m = 0 \\ 0, & m \neq 0 \end{cases} \quad m = 0, \pm 1, \ldots, \pm N.$$

Notice that we only enforce the zero-ISI condition on the $2N + 1$ samples. Why? We only have $2N + 1$ coefficients (variables) and can only achieve this much. Solve the $2N + 1$ equations and you obtain the zero-forcing equalizer.
Example:
A 3-tap equalizer

The equalizer cannot eliminate ISI beyond its span.
Eye Diagrams

- **Eye diagram**: Constructed by overlapping a *number of* segments of the base-band signals
- A qualitative measure of the system performance.

(Lee et al., Digital Comm., 1994)
Increasing bandwidth may mitigate ICI, but will also allow more noise to enter. Another example of trade-offs in communication systems design.

Figure 4.14
Eye diagrams for $B_N = 0.4$, 0.6, 1.0, and 2.0.

Amplitude jitter: ICI causes the amplitude of each symbol to fluctuate.

Timing jitter: ICI causes the timing of each symbol to fluctuate.

The best place to sample the signal.

Figure 4.15
Two-symbol eye diagrams for $B_N = 0.4$. 

Amplitude jitter: ICI causes the amplitude of each symbol to fluctuate.
Symbol, Bit, Word, and Frame

- Bits $\rightarrow$ Symbol (a single pulse in transmission) --- carrier sync., symbol timing
- Bits $\rightarrow$ Word --- word sync.
- Words $\rightarrow$ Frame --- frame sync.

(Benedetto et al., Digital Transmission Theory, 1987)
Synchronization

- Methods: (1) external sync signals;
  (2) **self-synchronization**: derivation from the modulated signals

- **Example 1**: squaring the received NRZ signals and PLL

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**Figure 4.16**
Simulation results for Computer Example 4.3. (a) NRZ waveform. (b) NRZ waveform filtered and squared. (c) FFT of squared NRZ waveform.
Example 2

Figure 4.17
System for deriving a symbol clock simulated in Computer Example 4.3

Figure 4.18
Simulation results for Computer Example 4.4. (a) Data waveform. (b) Data waveform multiplied by a half-bit delayed version of itself. (c) FFT spectrum of (b).
Carrier Modulation

- Simple modulation applied to baseband digital signals

Example: Let $d(t)$ be the NZR waveform.

Amplitude Shift-Keying (ASK): $x_{ASK}(t) = A_c[1 + d(t)]\cos(2\pi f_c t)$

Phase Shift-Keying (PSK): $x_{PSK}(t) = A_c \cos[2\pi f_c t + \frac{\pi}{2} d(t)]$

Frequency Shift-Keying (FSK): $x_{FSK}(t) = A_c \cos[2\pi f_c t + k_f \int_0^t d(\alpha) d\alpha]$
Figure 4.19
Examples of digital modulation schemes.